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Montecarlo DLA type simulation of non-wetting (Drainage) stable displacement in porous media*

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Abstract

We study immiscible fluid-fluid displacement in a porous media in a regime where the displaced fluid has negligible viscosity. In our model we consider the interplay between viscosity forces and capillary forces; the ratio is denoted by the parameter r, the inverse of the capillary number. We use a DLA type algorithm and consider a boundary condition at the interface of the two liquids, which takes into account the viscous, and the capillary pressure drop at the interface. This boundary condition makes the problem nonlinear. We make computer simulations and generate patterns of displacement. The roughness exponent α and the dynamic exponent β are calculated for each interface of the pattern obtained. We find that the roughness exponent depends on r and ranges from a value around 0.5 for small r to a saturation value 0.8 for large r. We also find strong fluctuations of these values during the simulations because of cascade similar processes at the interface. Our results compare well with the experiments of Pon-zeng Wong et. al. We further extend our model and introduce a parameter, which considers the relative preference of the wetting properties of the two liquids to the porous media. We find that this parameter controls the amount of trapped liquid behind the front.

Key words: fluid displacement; porous media.

Simulaciones Monte Carlo tipo DLA del desplazamiento de un fluido por otro en medios porosos para el caso estable y cuando el fluido desplazado moja preferentemente al medio poroso (Drainage)

Resumen

Estudiamos el desplazamiento inmiscible fluido-fluido en medios porosos, en el régimen donde el fluido desplazado tiene viscosidad insignificante. En nuestro modelo consideramos la interacción entre las fuerzas de la viscosidad y las fuerzas capilares, cuyo cociente es denotado en las simulaciones por el parámetro r y representa el inverso del número capilar. Utilizamos un algoritmo tipo DLA y consideramos una condición de borde en la interface de los dos fluidos

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(donde se toma en cuenta la viscosidad) y la caída de presión capilar en la interface. Esta condición de borde hace que el problema sea no lineal. Los patrones de desplazamiento se generan vía simulaciones en computador. Se usa la técnica de caminar por la interface cuyas fluctuaciones son medidas por medio del coeficiente de rugosidad (alpha) y el exponente dinámico (beta). Ambos se calculan en cada interface de los patrones de desplazamiento obtenidos. Encontramos que el exponente de rugosidad depende de r, variando desde 0.5 para valores pequeños de r hasta un valor de saturacion de 0.8 para grandes valores de r. También encontramos fuertes fluctuaciones de esos valores por efecto de procesos tipo cascada en la interface. Los resultados se comparan con resultados experimentales reportados por Pon-zeng Wong y colaboradores. Extendemos nuestro modelo e introducimos un parámetro que considera la preferencia relativa de adherencia o mojabilidad de los dos fluidos respecto al medio poroso. Encontramos que este parámetro controla la cantidad de fluido atrapado detrás del frente.

Palabras clave: Desplazamiento de fluidos; medios porosos.

Introduction

Basic research in displacement processes in porous media has attracted a number of relevant work recently in part because of the importance in the development of simulators for oil wells. In this work we investigate ideas concerning stable displacement. Stable immiscible fluid –fluid displacement occurs when a fluid of finite viscosity inside a porous media displaces another fluid of negligible viscosity. From an experimental point of view there are mainly two parameters, which govern the dynamics of the displacement: the capillary forces at the through level, and the viscous pressure drop on the displacing fluid. One important experiment, done by the group of Wong and coworkers (1) consisted in water displacing air in a porous media made of glass beads inside a Hele-Shaw cell. In that case the porous media was homogenously disordered. In a previous work we have considered the case of unstable displacement in porous media (2,4,5), i.e., where the displacing fluid has negligible viscosity. We constructed a model using a DLA (Diffusion limited aggregation) type model, which considers random capillary forces. Cooperative effects due to inter-porous surface tension were neglected as well as the effects due to preference wetting properties of the liquids. They were considered later in an attempt to explains the results of Stokes et.al. (4). We extend here our model developed in [4] for the case of stable displacement. First we develop the model in all its generality, however we show results only for the case where inter-porous surface tension or cooperative effects is negligible.

These effects are discussed for the case of unstable displacement in detail in other work.

Model

In experiments, one considers the injection of a fluid of viscosity μ_1 through a porous media filled with another fluid of negligible viscosity μ_2. Under suitable conditions the pressure drop is given by Darcy's equation:

$$v_i = -k_i \nabla_p \qquad [1]$$

where:

ν_1= Fluids velocities. k_i= relative permeability. $k_i = k / \mu_i$, k = permeability of porous media.

μ_i= viscosities. ∇_p = pressure gradient.

The fluids are incompressible, such that:

$$\nabla^2 p = 0 \qquad [2]$$

The last relation is Laplace's Equation which crucial for the development of the Montecarlo scheme. Now, in the diffusion limited aggregation process the random walk is given by the diffusion equation, the density of random walkers is given by:

$$\frac{\partial C(r,t)}{\partial t} = D\nabla C(r,t) \qquad [3]$$

D = (Diffusion constant)

The steady state situation is given by:

$$\frac{\partial C}{\partial t} = 0 \qquad [4]$$

$\nabla^2 C = 0$, i.e., the Laplace equation.

The model consists of Laplacian growth combined with a boundary condition at the interface. Laplacian growth is realized through a DLA type algorithm which incorporates through the interface boundary condition the pressure drop at the frontline. This drop is produced by the random capillary pressure drop in the throats which is represented by random numbers $\tilde{p}(r)$, where r defines a site at the interface and by $\tau(r)k(r)$. Here $\tau(r)$ is the random inter-porous surface tension and $k(r)$ is the curvature (3). There are two types of random walkers. Those which are released from the boundary far from the interface at the injection place and those which are released from the two kind of interfaces which develop due to the immiscibility of the fluids during the process of invasion. The front interface which separates the invading fluid from the displaced one and the interface behind the front which separates the invading fluid from the trapped fluid.

We define:

$$p' = p + \tau k_{\max} + \Delta\tilde{p}/2 \qquad [5]$$

where p is the pressure with $\nabla^2 p = 0$, and $p = p_0$ is the injection pressure at the the bottom of the cell, therefore $\nabla^2 p' = 0$ and by definition positive every where, $\nabla^2 \tilde{p}/2$ is the half width of the dispersion of the $\tilde{p}(r)$ numbers which are distributed between $-\Delta\tilde{p}/2$ and $\Delta\tilde{p}/2$, $\tau k_{\max}$ is the maximum value of the product of the surface tension and the maximum value of the local curvature at r. The curvature is $k = -(\Delta m \Delta L)/a_0$. Here $\Delta m = \pm 1$ if a site is added or removed and $\Delta L = \pm 1$ is the increment in the length of the interface boundary when a site is added or removed respectively (3). On the viscous side of the interface, i.e., on the invading fluid, one has:

$$p' = \tau k_{\max} + \tau(r)k(r) + \Delta\tilde{p}/2 + \tilde{p}(r) \qquad [6]$$

because $p = \tau(r)k(r) + \tilde{p}(r)$, we take $p = 0$ on the nonviscous side. In this way there is a jump in the pressure given by (7) as we go from the defennding fluid to the displacing one. The ratio of the probabilities for releasing walkers from the interface boundary to the bottom boundary is given by:

$$\frac{p'(r)}{p'_{bottom}} = \frac{P_{ib}(r)}{P_{bottom}} = \frac{\Delta\tilde{p}/2 + \tilde{p}_c(r) + \tau k_{\max}\tau(r)k(r)}{p_0 + \Delta\tilde{p}/2 + \tau k_{\max}} \qquad [7]$$

In order to avoid unnecessary random walking one releases the walkers from a line in principle but not necessarily just behind the front at a distance l far from the injection bottom. This probability is given by $P_l = \frac{a_o}{l} P_{bottom}$. One obtains from Equation [8] in the limit when in principle the bottom of injection is far away from the front, letting $l \Rightarrow \infty, p_0 \Rightarrow \infty$ but keeping the ratio p_0/l finite to ensure the displacing process at all:

$$\frac{P_{ib}(r)}{P_l} = r\left[q\left\{1 - \frac{2\tilde{p}(r)}{\Delta\tilde{p}}\right\} + \left\{1 + \frac{\tau(r)k(r)}{\tau k_{\max}}\right\}\right] \qquad [8]$$

where: $r = \dfrac{\tau k_{\max}}{\alpha_0 \langle \nabla p \rangle}$ and where $q = \dfrac{\Delta\tilde{p}/2}{\tau k_{\max}}$

In the limit when q >1 equation [9] reduces to :

$$\frac{P_i}{P_f} = \frac{r}{2}\left[1 + \frac{2\tilde{p}(r)}{\Delta\tilde{p}}\right] \qquad [9]$$

Here r measures the relative strength of the capillary forces to the viscous forces. Using Darci's law (equation [1]), the capillary number which is defined in experiments as

$$r = \frac{\Delta p\%}{a\langle\nabla p\rangle}$$

$Ca = {}^{U\mu}\!/_{\tau_0}$, where U is the velocity of injection, μ is the viscosity of the displacing fluid and τ_0 is the bare surface tension. One obtains:

$r \approx Ca^{-1}$

We first make simulations with the relation (10) when cooperative effects due to inter-porous surface tension are neglected. We use lattices LxL with L= 1024 sites to generate aggregates. Immiscibility requires that random walkers who would break the interface boundaries are not allowed. Figure 1 shows an aggregate for r=1000. One sees two important distinct features. The front interface boundary shows roughness at certain scales and there is trapped fluid behind the front. Once the cluster has reached around 70% we trace the front interface with an algorithm that walks along it and calculate its mean value and the fluctuations over this mean also called roughness

We carry out this procedure for a given length scale l_0.

$$\sigma(l_0) = \left\langle \left(\frac{1}{l_0}\sum_{i=1}^{L}(Y_i - Y)^2\right)^{1/2} \right\rangle \qquad [10]$$

Although there are overhangs, they make negligible contribution, such that $\sqrt{\sigma(l_0)}$ represents indeed the front interface width W. We make the standard log-log procedure to find possible scaling relations. In Figure 2 we plot $\log\sigma(l_0)$ versus $\log(l_0)$ for some values of r. One observes that there is a range of values l_0 below which there is scaling. i.e., a linear dependence whose slope increases with r. This slope is named the Hurst exponent H or roughness exponent α. The dependence of H with r describes well the experiment of Wong and collaborators, see Figure 3(d) of (1). Figure 3 shows H(r) for some values of r. This function is well fitted by $H(r) = a + b * \ln(r)$ for $r > 1$. For small r $H \approx 0.5$ and appears to saturate to $H \approx 0.8$ for larger values of r.

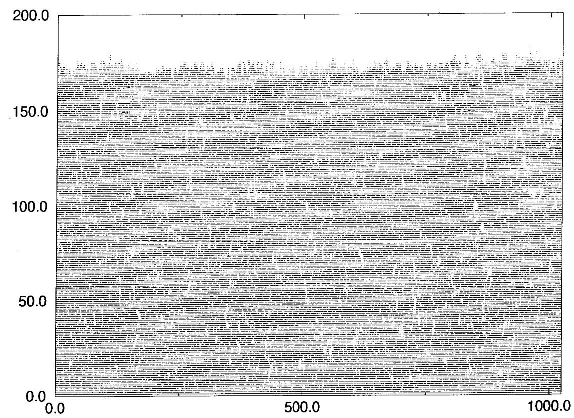

Figure 1. Typical cluster for immiscible stable displacement for L =1000 and r =1000.

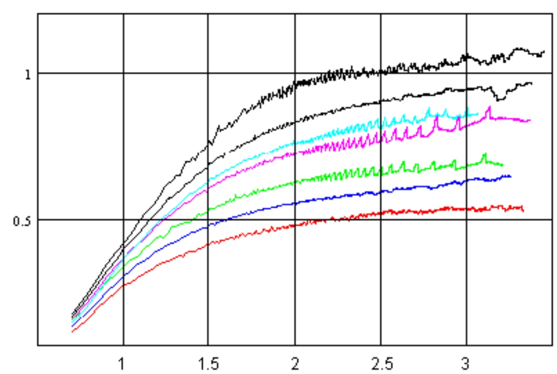

Figure 2. $\log\sigma(l_0)$ versus $\log(l_0)$ for different values of inverse capillary number r.

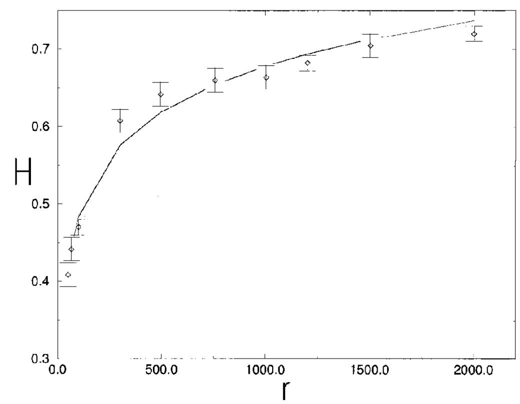

Figure 3. The Hurst exponent H as a function of the inverse capillary number *r*.

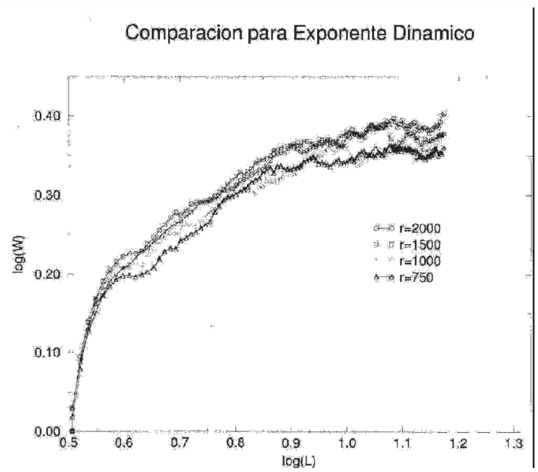

Figure 4. $logW(l_0, L(t))$ versus $L(t)$ for distinct values of r.

We also study the dynamic exponent β. For this purpose we study the temporal evolution of the width $W(l_0, L(t))$ from initial flat boundary conditions. Here t is a pseudo time which is measured as the mean height value of the aggregate L(t). Figure 4 shows how this width grows for distinct values of r for a given lengthscale l_0. Once notice the existence of two regimes of growth. One regime of rapid growth with $\beta = 2.82 \pm 0.06$ and another one following the first of rather slow growth with $\beta = 0.33 \pm 0.02$.

After this second regime of growth, saturation sets in and the stationary value of the width W for this particular r is reached. However, one sees from Fig.4, that the width fluctuates. The nature of these fluctuations appears to be related with cascade similar processes or intrinsic metastabilities of the front interface. Fluctuations in the width and therefore in the roughness exponent is also observed in (1), Figure 3b.

Conclusions

Our results show that the dependence of the roughness coefficient α on the capillary number r^{-1} for negligible inter-porous surface tension explains well the experiments of Wong et.al.. There are also strong fluctuations in this number because the existence of cascade similar processes at the front interface. We also found two regimes of growth with marked distinct values of β. This is a prediction for the experiments as Wong did not measure dynamic growth properties of clusters [6]. Furthermore our model contains more physics to be analyzed when inter-porous surface tension (cooperative effects in wetting displacement) can be relevant or relative wetting properties of the fluids can play an important role. Also the fluctuations in the width due to cascade similar processes are currently under investigation (7).

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