Opción, Año 35, Especial No.19 (2019): 2407-2422
ISSN 1012-1587/ISSNe: 2477-9385

Develop data encryption by using mathematical complement

Azhar malik
Computer Engineering Department / University of Technology / Baghdad / Iraq

Abstract:

(Global auto orphic induction for number fields). - Let's be the numbers fields, E has the cyclic extensions of F, of it is degree. Auto orphic inductions associate to the cuspidor auto orphic representations of GLm (AE) an auto orphic representation π of GLmd (AF), induce from cuspidor, & characterize by a fact that the place v of F, a factor L (πv, s) is the product of the factors L (Tw, s), where w run inside a place of E above v. By a correspondences conjecture by Langland's, that processes should corresponded to inducing Galois representation to E to F.
They proved here that a representation π auto morphically induce from T exist, & study a fiber and image of auto orphic inductions. For that use Clozel on basic changes, which correspond to Galois representation from F - E, and clarified a relation between two processes. And proved that global auto-orphic inductions are compatible, at finite place, and a local auto orphic induction defines by R. Herb & author.

Desarrollar cifrado de datos mediante el uso de complemento matemático

Resumen

Resumen:
(Inducción autofófica global para campos numéricos). - Seamos los campos de números, E tiene las extensiones cíclicas de F, de eso es grado. Las inducciones autofóricas se asocian a la escupidera con representaciones autofóricas de GLm (AE), una representación autofórica π de GLmd (AF), inducen desde la escupidera y se caracterizan por el hecho de que el lugar v de F, un factor L (πv, s) es el producto de los factores L (Tw, s), donde w corre dentro de un lugar de E arriba de v. Por una conjetura de correspondencias de Langland, que los procesos deberían corresponder a inducir la representación de Galois a E a F.
Demostraron aquí que existe una representación π auto inducida morfológicamente a partir de T, y estudian una fibra e imagen de inducciones autoorfas. Para ello, utilice Clozel en los cambios básicos, que corresponden a la representación de Galois de F - E, y aclaró una relación entre dos procesos. Y demostró que las inducciones auto-órficas globales son compatibles, en un lugar finito, y una inducción auto-órfica local la define R. Herb & author.

1. Introduction

1:1 Let F be the field of the numbers, E the cyclic extensions of F and d the degrees of E on F. Arthur and Clozel [1] established a process of change base, from F to E, which to a representation auto orphic cuspidate unitary π of GLn (AF) associates an auto orphic representation πE / F of GLn (AE), induced of unit cuspidate, so that in almost any place w of E, the factor L from πE / F to w is a producer of a factor L (χπv, s), where v is a place of F below w and where χ goes through the trivial F × v characters on the norms from E × w.
In ([1], chapter III), Arthur & Clozel determined image & the fibers of a bases exchange, at least when the degree d is a prime number. We want to study here the process in the opposite direction, called auto orphic induction, which at an auto orphic cuspidor unit representation T of GLm (AE) associates an auto orphic representation π = TE / F of GLmd (AF), induced of cuspidate unitary, so that for almost any place v of F a factor L (πv, s(a produced of a factor L (Tw, s), w traversing the squares of E above from v.

1.2 - â € "The existence of these two processes is easily predicted by the heuristic of Langlands, in which the auto orphic representations of GLn (AF) correspond To Galois representations of dimension n. To be more precise, the

auto orphic representations of GLn (AF) which are algebraic to infinite places in the sense of Clozel [2] must correspond to representations ad "abdicants of dimension n of the Gal (F / F) group - where F is the algebra closures F â € ", the auto orphic cusp representations corresponding to representations irreducible of Gal (F / F). If E is any affinity extensions from F to F, and from its degree, then the Gal (F on E) is the sub- "open group of index d of Gal (F / F). The restriction to Gal (F / E) of an "adic" representations of dimensions n of Gal (F on F) gives the n-dimensional representations of dimension n of Gal (F / E), while the induction at Gal (F / F) of an adic representations of dimensions m of the Gal (F / E) gives a representation of dimensionality md of Gal (F / F).
The basic change process for auto orphic representations of GLn (AF) is the counterpart of the restriction process of representations L – adic of Gal (F / F): the condition on the factors L accurately reflects the behavior of restriction to unbanked squares. Likewise the process of auto orphic induction is the counterpart of the process of induction of representations Galoisiennes, hence its name; the condition on the factors L reflects Also the behavior of Galois induction at unbranched places.
1.3. - It should be noted that the basic change is not established for all finite extensions E of F: the known cases E on F are cyclic makes it possible to treat where E is the Galois extensions of F to a solvable Galois's groups ([1], Chap. III), but the general case is for the moment out of reach. Similarly, the induction can only be established for the moment in situations successive cyclic inductions.
1.4. - For cyclic E / F the base change is obtained by comparison of two formulas of the traces, one for GLn (AF), the other for GLn (AE) but "twisted" by actions of a generator σ of the Gal (E on F). The induction auto orphic, always for cyclic E / F, must also be obtained by comparison two formulas of the traces, one for GLm (AE), the other for GLmd (A / F , (but "twisted" by the action of the characters of a×f / F× which defines extensions E on F. Moreover, it is by comparing such formulas with traces that local theory of basic change ([1, chapter. I]) and of auto orphic [5], induction were built. However the formulas of the traces of [5] are used in special cases where they have writing and a demonstration relatively simple. This is not the case for the formulas established in ([1], Chap. II) for basic change. We have moved back the task of using Arthur's later work: the auto orphic induction can to be interpreted in terms of endoscopy.
1.5. - More simply, we are deciding here the construction and the properties of the cyclic auto orphic induction, of those of the basic change. a point of view of a Langland heuristic, we use a procedure of "Galois descent" AT". More precisely, if it is a representation of dimensionality m of Gal (F / E), its induced Ï*f* to

Gal (F / F), which is of dimension m d, is stable by torsion by the characters of the Gal (E on F), & a restriction of Ï*f* to Gal (F / E) are a sum of a conjugate of Ï by the cyclic group Gal (E / F). In addition we can easily see that these two properties are characteristic. If Ï "is a cuspidal (unitary) auto orphic representation of GLm (AE), we will therefore try to define its auto orphic induction Ï = TE / F as the representation auto orphic of GLmd (AF), induced cuspidal unit, which is torsionally stable by the characters of A×F /F× defining E / F and whose change base to GLmd (AE) is the parabolic induced of gT g, g traversing Gal (E / F).

1.6. - To demonstrate the existence and uniqueness of π = TE / F, and obtain the properties of the process thus obtained, we need to know the fibers completely. and the image of the basic change. We know that the results of ([1], chapter III) in the case where d is prime do not allow to reach the general case by successive Galois descents of first degree [10], at least without work additional. This is why, at first, we take up the arguments of ([1], chapter III) to complete them by treating the case of an extension cyclic E / F of any degree. Then we deduce the case of induction auto morphed. We now state our results. We set the cyclic extensions E / F, from a degree d, the generator σ of the group Γ = Gal (E / F) and the generator κ of groups X from character of A× F trivia on F× NE / F (A×E).
1.7. - Let's start with the basic change. If π is a representation auto orphic of GLn (AF), induced by a unit cuspidate, we say that a representation auto orphic Π of GLn (AE), induced by unit cuspidate, is a change basis of π (from F to E) if the conditions on the L factors of (1.1) are verified. By the rigidity theorem of Jacquet and Shalika ([7], Theorem 4.4), Π is then unique to isomorphism, and we will talk about the change of base Π of π, that we can note πE / F. By the conditions of (1.1), we see that Π is stable by the action of Γ.

There is obvious compatibility with parabolic induction. If n1. . . nr are integers ≥ 1 of sum n, and that for i = 1,. . . , r, πi is a representation auto orphic of GLni (AF), induced by unit cuspidate, we note π1 × …… × πr the auto orphic representation of GLn (AF) obtained by parabolic induction of π1 …… πr. If for i = 1,. .. , r, Πi are the bases exchanges of πi, then Π1 × ….. Πr is a base change of π1 × ….. × πr.

1.8. - If π is the auto orphic representations of the GLn (AF), induced cuspidate unitary, we denote by X (π) its stabilizer in X, d (π) the cardinal of X (π). Of course d (π) divides d, but looking at the central characters we see that d (π) also divides n.
Let 1. - Let π be the auto orphic representations a unit cusp of GLn (AF), and let δ = d (π), n = δr. Then π has a basic change Π, which is of the form Π1 × Πσ1 × ·

ce of the student in particular as they are an important part of this program.
6. Taking advantage of the proposed strategic steps in this research which may contribute to the development of the practical education program in the Faculty of Education Mathematics Department and may contribute to the development of the skills of teaching mathematics for students applied.
Research Objectives: The current research aims to:
1 - Evaluatvior that is effective in achieving specific goals in the form of mental, verbal or motor responses that are done accurately and quickly and adapt to the teaching position. It is one of the
· · × Πσ δ-1 1, where Π1 is a representation auto orphic unitary cusp of GLr (AE), whose stabilizer in Γ is generated by σδ. The auto orphic representations of GLn (AF), induced by unitary cusp, whose base change is Π are the twisted of π by the characters of X.
This theorem implies that any auto orphic representation of GLn (AF), induced unit cuspidate, has a basic change. We describe in 2.5 the fibers of the process thus obtained.
1.9. - As for the image of the process of basic change, it is governed by the follow results.
2. Let Π be the auto orphic representations of the GLn (AE), of the form Π1 × …. × Πσδ-11, for a divisor δ of d and n, where Π1 is a unitary auto orphic representation of GLn / δ (AE), whose stabilizer in Γ is generated by σδ. Then Π is the basic change of a unitary auto orphic representation of GLn (AF).
In particular, we draw from it that any auto orphic representation of GLn (AE), induced by unit cuspidale, which is stable by σ, is a base change (2.5).
1.10. - Once acquired Theorems 1 and 2, we can then proceed by Galois descent to obtain the auto orphic induction.
If T is an auto orphic representation of GLm (AE), induced by a unit cuspidale, we say that an auto orphic representation π of GLmd (AF), induced by a unit cuspidal, is induced auto orphic by T (in the E / F extension). If the conditions on the L factors of (1.1) are satisfied. By rigidity, π is then unique to isomorphism, and we can speak of the auto orphic induced of T, and note it TE / F. By the conditions of (1.1), π is stable by torsion by κ.
Theorem 3. Let T be an auto orphic representation of GLm (AE), a unitary cuspidal inductivity. There exists an auto orphic representation π of GLmd (AF), induced of unit cuspidale, which satisfies the following conditions:
(i) A bases exchange of π is T × T σ × ….. × Tσd-1;
(ii) Π stability by torsion by κ.
The representation π is unique to isomorphism, and is induced auto orphic of T.
We will also describe the fibers and the image of the auto orphic induction process

thus obtained (2.7).

1.11. - As we recalled above, a local theory of cyclic base change is established in ([1], chapter I) for local bodies of zero characteristic; this version is extended to local bodies of non-zero characteristic in [6, ch. II].

More precisely, if L on K is the cyclic extensions of local bodies, the basic change theory, from L to K, associates to every class of isomorphism π of generic irreducible unit representations of GLn (K), a class of πL / K isomorphism of generic irreducible unitary representations of GLn (L). The representations π and πL / K verify a identity of characters, the identity of Shintani ([1], [chapter 1, def 6.1), which determines πL / K from π. Moreover, we know that by the Langlands correspondence, this process expresses the restriction to the Weil Deligne groups of L of an n dimensional representation of a Weil Deligne group of κ [3]. The theory extend in the simple way to the cases of the cyclic algebra L on a local body K (cf [9], § 9, and [6, chapter II]).

1.12. - Similarly, a theory of local auto orphic induction has been established for the cyclic extensions of non-Archimedean local bodies, or more generally for a cyclic algebra L on a non-Archimedean local field K ([5] and [6, Chapters III and IV]). More precisely, this theory associates to every class of isomorphism T of generic irreducible unit representations of GLm (L) an isomorphism class TL / K of generic irreducible unitary representations of GLmd (K), d is a degrees of L on K. The representations T & TL / K are connected by a character identity [5, § 4] which determines TL / K from T. Moreover, when L is the cyclic extensions of K, this process corresponds well, by the correspondence of Langlands, to the induction of K to L representations of dimension m of the Weil-Deligne group of L [3, 4].

1.13. - It is then necessary to study the links between local processes of basic change and auto orphic induction, as well as the compatibility of these local processes with global processes.

With regard to the base change, the local-global compatibility is known if d is a prime number ([1], chapter I, page 69 and chapter III § 5). We extend the arguments of loc. cit. to obtain the follow theory:

Theory 4. π be the auto orphic representations of GLn (AF), induced by a unit cuspidal, & let Π be its basic change from F to E. Let v be the places of F, & let us see the algebra Ev = E F Fv as a cyclic Fv-algebra of group Γ. Then the representation Πv of GLn (Ev) is the base change of πv.

Note. - The last assertion is equivalent to the fact that for every place w of E through v, πw are the base change of πv, for the cyclic extension Ew / Fv.

1.14. - We also demonstrate the analog of this result for auto orphic induction.

Theorem 5. - Let T be an auto orphic representations of GLm (AE), induced by a unit cuspidale, & let π = TE / F be a auto orphic induced of T. Let v be the finite places of F. Then πv is a automorphic induced representation of the representation Tv of GLm (Ev).
Note. - 1) for a global body of nonzero characteristic, statements similar to theorems 4 and 5 are established in [4].

2) The proof of Theory 5 extends directly to a case of an infinite place of F, as soon as the theory of auto orphic induction established for the Archimedean bodies [4].

1.15. - In fact, given the construction of global auto orphic induction, Theorem 5 will derive from Theorem 4 & a compatible of local bases exchange and auto orphic induction processes.
Theory 6. Let v be the affinity places of F, & let T be the irreducible, generic and unitary smooth representations of GLm (Ev). Then there exist smooth irreducible generic unit π representations of GLmd (Fv) which satisfies the following properties:
(i) the base change of π (for the cyclic algebra Ev / Fv) is T× Tσ × ….. Tσd-1.
(ii) Π is stable by Kv torsion.
The representation π is unique to isomorphism, and is induced auto orphic of T.

1.16. - Basically we show our results in the order shown. The chapter 2 we generalize the considerations of ([1], chapter III) to the cyclic case of degree not necessarily prime, to obtain the theorems 1 and 2 and the general description of the image and the fibers of the base change. We also derive the theorem 3, and describe the image and the fibers of the auto orphic induction. Chapter 3 is devoted to the proof of Theorems 4 to 6.
The reader will have noticed, as the rapporteur has noted, that we have not recalled the precise character identities that determine local processes of basic change or auto orphic induction. The point is that these relationships do not serve us here. In the arguments of chapter 3, of a local-global nature, we only use that these processes can be defined locally, and in certain circumstances already established ([1, Chapter I], [5]), they are compatible with global processes.

1.17. - This article is written during a research leave granted by Paris-Sud University I. Badulescu, L. Clozel, B. Lemaire and F. Shahidi for discussion on this article. the reporters for his insightful remarks.

1.18. - In the following, we keep the previous situation. So F is a body of num

bers, E a cyclic extension of F, of degrees d. put Γ = Gal (E / F) & fix a generate σ of Γ; we denote X the group of trivial A ×F characters on F × NE / F (A× E) and we fix a generator κ. We note | | F the absolute adelic value on AF.
If v place F, put Ev = E F Fv; it is the product of the complemented Ew when w traverses the places of E above w, and it is a cyclic Fv-algebra of group Γ. The usual notation is used for non-Archimedean local bodies.
Integer's m and n, strictly positive, are generally considered fixed. The auto orphic representations we consider are generally unitary, but we always specify this point, for the convenience of the reader.

2. Basic change and automorphic induction
2.1. -   In this chapter, we use the arguments of ([1], chapter III) for prove the theorems 1 and 2. These arguments have two aspects: a comparison of formulas of the traces on the one hand, and considerations of poles of functions L of pairs on the other hand. While these two aspects are intertwined in [loc. cit], we strive here to separate their role, which, in addition to the advantage of clarity, makes it possible to generalize the results of [loc. cit.] at any level d.
We mainly consider here automorphic representations induced of unit cuspidale in a sense of [loc& cit., chap. II, and def. 4.1]. Recall that an auto orphic representations π of GLn (AF to be induced as a unit cuspidale if it is obtain by parabolic inductions of the cuspidal unitary automorphic representations T of a Levi sub group of GLn. like that subgroup of Levi is isomorphic to GLn1 × ….. × GLnr, where n1 + ….. + nr = n, and T is of the form π1   ….   πr where for i = 1,. . . , r, πi is an auto orphic cuspid representations of GLni (AF). in this case π = π1 × …. πr and we say that π1   ….   πr is the cuspidal supported of π; this cuspidal support are well defined to isomorphism and permutation of factors [loc. cit., chap. III, 2.4]. This uniqueness result stems from considerations of L-functions of pairs, and the same reasonings also give the phenomenon of rigidity: if Ï € ï¿¿ is an automorphic representation of GLn (AF), induced of unit cuspidale, and such that Ï € ï¿½ is isomorphous for almost every place in F, then it is isomorphic to it.
Note. Â € "In a places v of F, the € components of Ï generic and unit. Ecrivant π = π1 ×· · ·× πr as above, πv is the parabolic induced π1,v ×· · ·×πr,v, which we know is irreducible. By [8], we see that in order for the stiffness phenomenon ci- above to be valid, it is not necessary that the automorphic representation π be induced from cuspidale unit.
Note that for finite squares, the classification of generic irreducible smooth representations is due to Zelevinsky [13, § 9], and that of unitary irreducible smooth representations to Tadić [12].

2.2. - However, the automorphic representations that can intervene in the formula of the traces - or rather its discrete part, which is the only one which plays a role in ([1], chapter III) - are not all induced of cuspidale.
We will say that an auto morphic representations π of GLn (AF) is discrete if it occurs in a discrete part of L2 (GLn (F) \ GLn (AF), ω) for a unitary characters ω of trivial A×F on F×. Moeglin and Waldspurger [11] classified discrete representations in terms of automorphic cuspidales unitary representations. if π is a discrete auto morphic representations of GLn (AF), there exist the divisor r of n, n = rs, & a representations automorphic cuspidal unit σ of GLr (AF), like that at any places v - F a component πv is the only irreducible quotiention of a parabolic induced
ρv | | v(s−1)/2 ×· · ·×ρv | |v(1−s)/2 .

In particular, if v is finite and Pv is unbranched, parametrized by a diagonal matrix Av of GLr (C), then πv is parametrized by the matrix of GLn (C) diagonal block Av q v(1-s) / 2,. . . , Av q v (1-v) / 2 (here | |v denotes the norm of Fv, and qv the cardinal of the residual body of OFv). Note that all the local components of ρ are generic and unitary, which in the previous case implies that the eigenvalues α of Av satisfy q v −1/2 < |α| < q v 1/2:
We see that the integer r is determined by π. But arguments of functions L of pairs prove that π also determines ρ to isomorphism. It also follows that the discrete representations verify the phenomenon of rigidity.
In fact, the automorphic representations that occur in discrete parts of a trace formula of ([1], chapter II) are induced discrete series: replace, in the definition of 2.1, "unitary cusp" by "discrete". The discrete series inductors also verify the phenomenon of rigidity.

2.3. - We prove the theorems 1 and 2 by inductions on n, a case n = 1 arising from the global theory of the class field.
First, an auto morphic representations π of the GLn (AF) induced by the unit cuspid has a basic change: for π cuspidale, a case where one immediately restricts oneself, this is proved in [loc. cit., chap. III, Thm. 4.2]. The demonstration pp. 203-204 (proof of Thm 4.2 (a)) does not use that d is prime: the important point is that because of the rigidity phenomenon π intervenes in the trace formula for GLn (AF) with a non coefficient no. (Of course, we can obtain the result for d any from the case where d is prime, by successive extensions of first degree, but that would not be valid for the case that follows).
In a opposite directions, let Π be the automorphic representations of GLn (AE)

of the form Π1 × Πσ1× · · · × Π1 σδ−1 where δ is a divisor of d and n, Π1 Being a unitary automorphic cpepal representation of GLn/δ (AE), of stabilizer in T "generated by σδ. So the proof of Thm. 4.2 (e) in [loc. cit., pp. 207-209] applies and gives that π is a basic change: just take δ Instead of the L whole [loc. cit.].

Note. - At the beginning of their proof, Arthur and Clozel note that Π1 …. Π1σδ-1 gives the non zero contributions to a formula of twisted traces. Il It should be noted that due to the phenomenon of rigidity this contribution cannot be canceled by any other.

2.4. - now completed a proof of Theorem 1 and 2, using as in [loc. cit.] the functions L of pairs. Remember that X is the groups of character in A F× trivialed on F × NE / F (A E×).

Let π be first an auto morphic representations of the unit cusp of GLn (AF) and let δ = d (π), n = δr. Let Π be a basic change of π and Π1 …. Πt its cuspidal support; as Π is stable by σ, Πσ1 …. Πσt is isomorphic to Π1 …. Πt, to order near the factors. Let ε be the divisor of d such that the stabilizer of Π1 in Γ is generated by σε; we can then assume Πi = Π1σi−1 pour i = 1, …, ε . Il is to prove that ε = δ = t. Suppose first ε < t. So Π/ = Π1×…. × Π1σε−1 is, by the inverse hypothesis, the basic change of an automorphic representation cuspidale Unitary π' of GLn, (AF) for an integer n' < n. If S is the finite group of squares of F, large enough, it is equal

LS(Π × Π' , s) = Πχ X LS(χπ × π' , s).

But the left factor has a pole of order at least ε in s = 1, while the one on the right does not have one. So we have ε = t, and we then consider equality LS (Π ×Π , s) = Πχ X LS(χπ×π , s): the left-hand side have the pole of order ε in s = 1 and a right one the pole of order δ, hence ε = δ.

Let π' be the auto morphic representations of GLn (AF), induced unit cuspidale, & of the same basic change as π; is π'1 …. π't the cuspidal support of π'. So Π is isomorphic to Π'1 ×…× Π't or for i = 1, . . . , t Π'I are the basic change of π'I; looking at the action of Γ, it already implies t =1 : so π' is cuspidale and equality

LS(Π × Π , s) = Πχ X LS(χπ × π' , s).

Leads that π' are of a form χ π for a χ X. Of course each χ π has for base change Π. This proves Theorem 1.

2.5. - Il We have yet to completed a proof of Theory 2. And its ratings, we have already 2.3 what is the changeover of an automorphic presentation π of GLn (AF) induced unit cuspidale. But then, considering the above, the fact that the cuspidal support of Π forms a single orbit under Γ implies that π is cuspidal.
The same arguments make it possible to described an image & a fi bres of a basic exchange, for the induces of unit cuspids. Here is the result.
Proposition.-An automorphic representation Π of GLn (AE), induced by a unit cuspidale, is a base change if and only if its cuspidal support is stable by Γ. Yes π = π1×···×πr, where the πi
are unitary cuspids, a Π for base change, then the other automorphic representations of GLn (AF), induced by a unit cuspidale, having Π for basic change, are the χ1π1 ×···×χrπr, where the χi travel X .

2.6. - We are now able deprouverlethéorème 3. Let T be the auto morphic representations of the GLm (AE) induced unit cuspidale, and either T1 ··· Tr its cuspidal support. For i= 1, …, r, let δi be the divisor of d such that the stabilizer of Ti in Γ is generated by σδi, and ask d = δi si. So the cuspidal support of Π=T ×Tσ×···×Tσd−1 is formed of the Ti orbits in Γ, each repeated si time. If πi is this automorphic cuspidale unitary representation whose base change is Ti ×Tσ i ×···×Tσδi−1 i , the automorphic representations π of GLmd (AF), induced of unit cuspidale, whose base change is Π are those which have a cuspidal support of the form
χ11 π1 ··· χ1s1 π1 χ21 π2 ··· χ2s2 π2 ··· χr1 πr ··· χrsr πr
Where the χij, 1≤i≤r, 1≤j ≤si are in X.

To impose that π is stable by X amounts to imposing that its cuspidal support is, in other words that each χπi for χ X appear as many times as πi. As the stabilizer πi in X
has for cardinal δi, it is clearly possible, and determines the cuspidal support of π to the order.
Let v be a finite places to F like that πv is unbranched. Since a basic change of π is T ×Tσ ×···×Tσd−1, we have
Π χ XL(χvπv,s)= Π w|vL(Tw ×Tσw ×···×Tσd−1w)= Π w|vL(Tw,s)d
And each of the representations Tw is unbranched. Since π is stable by κ twist, we deduce L(πv,s)= Π w|vL(Tw,s). This proves that π is induced automorphic of T.

2.7. - As above, we now draw the image and the fi bers of the process of basic change.
Proposition.-An automorphic representation π of GLmd (AF), induced by a unit cuspidale, is an automorphic induction if and only if it is stable by X. It is the automorphic induction of a unitary cusp representation of GLm (AE) if and only if its cuspidal support forms a single orbit under X. if T1,...,Tr are automorphic representations of unit cuspids such that π is the automorphic induced of T1 ×⋯×Tr, then the automorphic representations of GLm (AE) whose automorphic induction is π are those whose cuspidal support has the form Tg11 ×⋯× Tgrr with g1, …,gr in Γ.

Demonstration. - Stability by X is certainly a necessary condition. To prove that it is suf fi cient, it suffices to prove a second assertion. Let π be the auto morphic representations of GLmd (AF), induced unit cuspidale, whose cuspidal support forms a single orbit under X. Il There is therefore a divisor si of d and a unitary automorphic cuspidale unit πi of GLmd / si (AF), whose stabilizer in X is generated by κsi, such that π is isomorphic to π1 × κπ1 ×⋯×κsi−1π 1.
As d (π1) =d/si = δi, the base change de π1 is of the form Π1×⋯× Π1 σδi−1 where Π1 is a unitary automorphic cusp representation, destabilizer in Γ generated by σδi. The change in the basis of ka π1 being the same, that of π is good Π1 ×⋯×Π1 σd−1 and π is the automorphic induced of Π1. Il It remains to deduce the fibers of the automorphic induction, i.e. the last assertion of the proposition. Certainly, by construction, the representations of the form T1g1 ×⋯× T r gr have even induced automorphic T1 × ⋯ × Tr. But the cuspidal support of π base change is
T1 T1 σ ⋅⋅ T1 σd−1 T2 ⋯ T2 σd−1 ⋯ Tr ⋯ T r σd−1,
& if, with obviousnotations, π is alsoinducedfrom T1' × ⋯ × T's then a cuspidor support of a basic change of π is also
T'1 T1 'σ ⋯ T1 'σd−1 T'2 ⋯⋅ T2 'σd−1 ⋯⋅ T's ⋯⋅ T s 'σd−1;

It follows easily that s = r and that, with permutation close T'1 ⋯ T's is of the form T1g1 ⋯ Trgr with the gi in Γ.

3. Compatibility

3.1. - This last chapter is devoted to the proof of theorems 4 to 6. Theorem 4 is demonstrated in ([1], chapter III, Thm 5.1) in the case where d is prime. If d is

not prime, but π is a unitary auto morphic representations of GLn (AF) like that d (π) = 1- which is equivalent to the fact that a basic change Π of π is still cuspidor - then the demonstration of [loc. cit. p. 212] transposes without change and gives the result. We reduce the general case to those already known cases.
For this, we notice that by construction the global base change is transitive: if F' is the intermediate extensions between E and F, a πE / F (πF' / F) E / F' for every automorphic representation π of GLn (AF), induced unit cuspidale. It suffices to see that the same is true of the local base change, the general case of Theorem 4 then arising, by successive cyclic extensions of the first degree, from the case where d is prime. (Of course, once theorem 4 has been established in general, the transitivity of a local base change, at least for a local component of auto morphic representation induced by the unit cuspidale, follows). Unfortunately, the transitivity of the local base change is not explicit in ([1], chapter I). It should therefore be established here. It is also very useful.

3.2.
Proposition.-Let K is a locally compact commutative body of zero characteristic, & let L be a cyclic extensions of K, of group Δ. Let Δ' be the subgroup of Δ, and K' be the subfield of L fi xed by Δ'. Let ρ be the generic irreducible smooth representations of GLn (K), ρ' its base change from K to K', ρ'' a base exchange of ρ' from K' to L. Then ρ'' is also a basic exchange of ρ from K to L.
An Archimedean case is clear: we then have | Δ | ≤2. assume that K is non Archimedean.
The representation ρ is of the form ρ1 × ⋯ × ρr, where ρi is essentially square integrable, ρi = ρi◦ | |K8i with ρ i ◦ of integrable square and | si | <1/2. If, from K to K' , ρi◦ has a base change ρi10 (which is temperate), then ρ a for a base change ρ' = ρi10| | L s1 × ⋯ × ρr10| | L sr ([1], chapter 1, § 6.4) and we have a similar result for p'' . We are therefore reduce to a case where p represent square integral. We can even assume that ρ is a unit cuspidale ([1], chapter I, lemma 6.12).

We can then choose the cyclic extensions E/F of number field, of degrees [L: K], with a finite v place of F giving an isomorphism ι of Ev / Fv with L / K. We can also choose a unitary automorphic representation T of GLn (AF) whose component in v corresponds to ρ via ι. We can even suppose that the component of T in finity places of F split in E on F is cuspidal; this implies that the basic change from T to E, or to any intermediate extension, is cuspidal. Let us denote the base change from T to F', where F' is a intermediate exten sions of E / F fi xed by Δ' - note that E / F is the cyclic extension of group Δ -, and note T'' the change of base of T' to E, which is also the basic exchange from T to E. By the already known case of Theorem 4, T' v is a basic exchange of Tv in F' v / F''v, T'v that of Tv in Ev / Fv, and also that of Tv in Ev / Fv. Transporting via ι, we obtain the proposition, which also ends the proof of Theorem 4.

3.3 - Let's go on to the proof of Theorem 6. Note that this is a local statement: we could have replaced Ev/Fv by any cyclic algebra L/K, of group Γ, on the non Archimedean local commutative fields K (of zero character). use this notation & put η = κv.

The cyclic algebra L / K appears as a body product L' × σ L' ×· · ·× σe−1 L', where e is a divisor of d, d= ed ' , and or L' is the cyclical extensions of K , group generated by σ ' = σe. The character η defines the extension L'/K, which are of degrees d ' .

Let T be the unitary irreducible smooth representations of GLm (L); there exist unitary unit irreducible smooth representations T1,. . . , Te of GLm(L'), such that T is isomorphic to T1 T2 ◦σ−1 · · · Te ◦σ1−e. Say that a basic change of π, for cyclic algebra L/K, is

T ×Tσ×· · ·× Tσd−1 means that the base change of π, for the cyclic extension L'/K, Is

(T1×Tσ'1 ×…×Tσ'd'−11) × (T2×Tσ'2 ×…×T2σ'd '−1) ×..× (Te×Tσ'e ×…× T e σ'd ''−1) .

To proveed a existence & uniqueness of π, we are thus reduce to a case where L is the cyclic extensions of K. On the other hand the automorphic induced TL/K of T (for cyclic algebra L / K) is nothing else that T1L'/K × T2L'/K ×· · ·× TeL'/K , so that to prove the equality π = TL / K we can also assumes that L is the cyclic extensions of K.

3.4. - It is therefore assumed that L / K are a cyclic extension. The existence and the uniqueness of π then flow, as in the proof of Theorem 3, from the knowledge of the image and the fibers of the local base change, for the unitary irreducible smooth representations of GLm (K). Since Theorem 6.2 and Proposition 6.7 of ([1], Chapter I) deal only with temperate representations, we must now complete the arguments.

Proposal. - L / K are assumed to be a cyclic extension of non-Archimedean local bodies.

(a) Any smooth generic unit irreducible representation of GLn (L) that is σ- stable is the base change of unitary irreducible smooths representations of GLn (K).

(b) Let ρ the smooth irreducible generic unitary representations of GLn (K), of a form P1 × ⋯ × Pr where the Pi are essentially integrable square. The unitary unit irreducible smooth representations of GLn (K) having the same base change as P are the representations χ1ρ1 × · · · × χrρr, where the χi travel through the powers of η.

3.5. - Let's prove this proposition. It is clear that a basic change is σ- stable. Conversely, let T be the unitary irreducible smoothes representations of GLn (L), which we write T= T1 × · · · × Tr where the Ti are essentially square integrable. If T is σ- stable, we can suppose that the first Ti, say T1,. . . , Tk, form the orbit of T1 under the action of σ; if k <r, then Tk + 1 × · · · × Tr is also σ- stable. To prove that T is a base change, we can assume that Ti form a single orbit under σ. By torsion by an unbranched character, we then reduce to a case where T is tempera, which is given by ([1], chapter I, Thm6.2). This gives (a). Now prove (b). It is clear that the above representations have the same basic change as ρ. Write Pi = Pi◦ | |Ksi with integrable square Pi◦, and let Ti◦ be the base change of Pi◦, which is tempered. A base exchange of P is then T1◦ | | Lsr × … × Tr◦ | | Lsr. If a generic irreducible smooth representation and unitary P' of GLn (K) has the same base change as P, we write P ' in the same way, so with obvious notations we have

T1◦ | | Lsi × … × Tr◦ | | Lsr T1'◦ | | Ls'1 × … × T 'r '◦ | | Ls 'r ' .

We then separate according to the different values of si and s'i to be reduce to a case let P is tempered, which is given by [loc.cit. Chap. I, proposed. 6.7].

3.6. - The same arguments as for Theorem 3 thus give the existence and the uniqueness of π in Theorem 6. It remains to prove that π is the automorphic induced of T. We can continue to assume that L / K are an extension cyclic. We come back to the case where T is integrable square.

It is then used that for integrable square T, the automorphic induced of T is constructed in [5] by a global method. can assume that L on K is a cyclic extensions Ev / Fv - we assume, to conform to [5], that E / F is deployed at infinite places - and that T is the local component in V of a representation automorphic Cuspidal Unit T of GLm (AE) which satisfies the other properties imposed by ([5], § 8). Then TL / K are the component in V of the automorphic representation Π of GLmd (AF), induced of unit cuspidale, which is the automorphic induction of T. By Theorem 3, the basic change to F from E of Π is T × Tσ × · · × Tσd-1. Looking instead at V, we see that the base change of TL / K, for the extension L / K, is none other than T × Tσ × · · · × Tσd-1. Since TL / K is torsionally stable by η, we have TL / K = π.

3.7. - It remains for us to prove Theorem 5. But it is an immediate consequence of the construction of the global auto orphic induction given by Theorem 3, and Theorem 6.

Note. - We have established [4] the local auto morphic inductions in a Archimedean case, & proved the equivalent of Theorem 6. Theorem 5 is therefore valid for infinite places.

Reference

[1] J. Arthur & L. Clozel - Simple algebra, basic change, and the advanced theory of the trace formula, Annals of Math. Studies, vol. 120, Princeton Univ. Press, 1989.
[2] L. Clozel - "Patterns and automorphic forms: applications of the principle of functoriality", in Automorphic forms, Shimura varieties, and L-functions, Vol. I (Ann Arbor, MI, 1988), Perspect. Math., Vol. 10, Academic Press, 1990, p. 77-159.
[3] G. Henniart - "On the local Langlands conjecture for GLn", J. Theor. Numbers Bordeaux 13 (2001), p. 167-187.
[4] ____, "Automorphic Induction for GL (n, C)", J. Funct. Anal. 258 (2010), p. 3082-3096.
[5] G. Henniart & R. Herb - "Automorphic induction for GL (n) (over local non-Archimedean fields)", Duke Math. J. 78 (1995), p. 131-192.
[6] G. Henniart & B. Lemaire - "Basic change and automatic induction for GLn in non-zero characteristic", Memoirs of the S.M.F. 124 (2011).
[7] H. Jacquet & J. A. Shalika - "OnEuler products and the classification of automorphic forms. II, Amer. J. Math. 103 (1981), p. 777-815.
[8] R. P. Langlands - "On the notion of an automorphic representation", Proceedings of Symposia in Pure Mathematics 33 (1979), p. 203 207.
[9] _________, Base change for GL (2), Annals of Math. Studies, vol. 96, Princeton
Univ. Press, 1980.
[10] E. Lapid & J. Rogawski - "On twists of cuspidal representations of GL (2)", Forum Math. 10 (1998), p. 175-197.
[11] C. Moeglin & J.-L. Waldspurger - "The Residual Spectrum of GL (n)" Ann. Sci. School Norm. Sup. 22 (1989), p. 605-674.
[12] M. Tadić - "Classification of unitary representations in irreducible representations of general linear group (non-Archimedean case)", Ann. Sci. School Norm. Sup. 19 (1986), p. 335-382.
[13] A. V. Zelevinsky - "Induced representations of reductive p-adic groups. II. It is irreducible representations of GL (n) ", Ann. Sci. School Norm. Sup.13 (1980), p. 165-210.