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Revista de Antropología, Ciencias de la Comunicación y de la Información, Filosofía, Lingüística y Semiótica, Problemas del Desarrollo, la Ciencia y la Tecnología

Año 35, mayo 2019 Nº 89

Revista de Ciencias Humanas y Sociales

ISSN 1012-1587 ISSNe: 2477-9385

Depósito Legal pp 198402ZU45

Universidad del Zulia

Facultad Experimental de Ciencias

Departamento de Ciencias Humanas

Maracaibo - Venezuela

Opcion, Año 35, Nº 89 (2019):*2899-2921*
ISSN 1012-1587/ISSNe: 2477-9385

Estimating Fuzzy hazard Rate Function of Generalized Exponential Distribution

Nathier A. Ibrahim

Al – Turath University
irwsno@turath.edu.iq

Abstract

the paper deals with estimating two parameters (β,α) of generalized exponential failure model, then comparing fuzzy hazard rate function model, the methods of estimation are, moments, maximum likelihood, and proposed one, depend on frequency ratio method were it is derived according to studied distribution, then used for estimation parameters (β,α).

Keywords: Fuzzy two parameters Exponential Model (FEM), Fuzzy Hazard rate, Moment Estimator, MLE, Proposed one.

Estimación De La Función De Tasa De Riesgo Difusa De La Distribución Exponencial Generalizada

Resumen
El artículo trata de estimar dos parámetros (β, α) del modelo de falla exponencial generalizada, luego compara el modelo de función de tasa de riesgo difuso, los métodos de estimación son, los momentos, la probabilidad máxima y el propuesto, dependen del método de relación de frecuencia donde se deriva de acuerdo con la distribución estudiada, luego utilizada para los parámetros de estimación (β, α).

Palabras clave: Modelo exponencial de dos parámetros difusos (FEM), tasa de riesgo difuso, estimador de momento, MLE, uno propuesto.

1. Introduction

The concept of reliability and hazard rate function are probabilities that represents the life time (which is random variable) from beginning of a failure, after which we need further operation to modify the production operation. The widest application of reliability and hazard rate function are extended now due to introducing fuzzy factors on these function [R_X (x)H_X (x)], so we work on comparing different fuzzy hazard rate function estimators of generalized exponential distribution, since the hazard rate function is used widely in a variety of business and industrial setting. Many researchers indicate the estimation of reliability of system and distribution like, Bhattachargy and Johnson (1974) studied the reliability of multi component system. In (1981)[9], Kim and Kang studied the estimation of scale parameter of multi component stress – strength for the Weibull distribution with unknown scale parameter, while in (1991)[12] Pandy and Uddin work on estimating reliability of multicomponent of stress – strength model based on Burr distribution, they compare Bayesian and non-Bayesian estimator. In (2016)[2] Abass and Rana work on estimating the shape parameter of generalized exponential distribution when the scale parameter is known using Shrinksge estimator and compare the results numerically, also in (2016)[13], Rao et al estimated the reliability system in a multicomponent stress – strength with application on exponentiated Weibull distribution.

2. Theoretical Aspect

Let (x) be continues random variable follows two parameters generalized exponential with (α) as shape parameter and (β) is the scale parameter with p.d.f;

$$f(x,\alpha,\beta) = \alpha\beta e^{-\beta x}\,(1-e^{-\beta x})^{\alpha-1} \quad x,\alpha,\beta > 0 \tag{1}$$

While the CDF is;

$$F(x,\alpha,\beta) = (1-e^{-\beta x})^{\alpha} \quad x,\alpha,\beta > 0 \tag{2}$$

The Reliability function is;

$$R(x,\alpha,\beta) = 1-(1-e^{-\beta x})^{\alpha} \tag{3}$$

While the hazard rate function is;

$$h(x,\alpha,\beta) = \frac{f(x,\alpha,\beta)}{R(x,\alpha,\beta)} = \frac{\alpha\beta e^{-\beta x}(1-e^{-\beta x})^{\alpha-1}}{(1-e^{-\beta x})^{\alpha}} \tag{4}$$

The rth moments about origin is;

$$\mu_r' = E(x^r) = \int_0^\infty x^r f(x,\alpha,\beta)dx$$

$$= \alpha\beta^{-r}\Gamma(r+1)\sum_{i=0}^{\infty}(-1)^i C_i^{\alpha-1}(i+1)^{-(r+1)} \tag{5}$$

$$E(x) = \frac{1}{\beta}[\psi(\alpha+1)-\psi(1)] \tag{6}$$

$$v(x) = \frac{1}{\beta^2}[\psi'(1)-\psi'(\alpha+1)] \tag{7}$$

Were $\psi(\alpha)$ is called digamma of (α) and;

$$\psi(\alpha) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}$$

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$$

$$\Gamma'(\alpha) = \frac{d\lambda(\alpha)}{d(\alpha)}\int_0^\infty y^{\alpha-1}\ln y\, e^{-y}dy \tag{8}$$

3. Estimation Methods

The two parameters (α,β) are estimated by method of moments , then maximum likelihood, and percentile one, $(\hat{\alpha},\hat{\beta})$ are used to compare different fuzzy reliability function.

3.1 Moment Estimators

The $(\hat{\alpha}_{mom}, \hat{\beta}_{mom})$ are obtained from solving;

$$E(x) = \frac{\sum_{i=1}^{n} x_i}{n} = \overline{x}$$

$$E(x^2) = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

Then;

$$\overline{x} = \frac{1}{\beta}[\psi(\hat{\alpha}+1) - \psi(1)]$$

$$\frac{\sum_{i=1}^{n} x_i^2}{n} = \frac{1}{\beta^2}[\psi'(1) - \psi'(\alpha+1)] + [E(x)]^2 = v(x) + [E(x)]^2$$

$$\overline{x} = \frac{1}{\beta}[\frac{\Gamma'(\alpha+1)}{\Gamma(\alpha+1)} - \psi(1)]$$

$$\hat{\beta}_{mom} = \frac{\lambda'(\beta_0+1)}{\lambda(\beta_0)}[\hat{\beta}\overline{x} + \psi(1)] \quad (9)$$

$$\hat{\alpha}_{mom} = \frac{\Gamma'(\alpha_0+1)}{\Gamma(\alpha_0)[\hat{\beta}\overline{x} + \psi(1)]} \quad (10)$$

Where (α_0, β_0) are initial values.

3.2 Frequency Ratio Method

This method depend on simplifying the ration of $(\frac{f_1}{f_2})$ as;

$$\frac{f_1}{f_2} = \frac{\alpha\beta e^{-\beta x_1}(1-e^{-\beta x_1})^{\alpha-1}}{\alpha\beta e^{-\beta x_2}(1-e^{-\beta x_2})^{\alpha-1}} = e^{-\beta(x_1-x_2)}\left(\frac{1-e^{-\beta x_1}}{1-e^{-\beta x_2}}\right)^{\alpha-1}$$

$$\ln\left(\frac{f_1}{f_2}\right) = -\beta(x_1 - x_2) + (\alpha-1)\left[\ln\left(1-e^{-\beta x_1}\right) - \ln\left(1-e^{-\beta x_2}\right)\right]$$

$$\beta(x_1 - x_2) = (\alpha-1)\ln\left(\frac{1-e^{-\beta x_1}}{1-e^{-\beta x_2}}\right) - \ln\left(\frac{f_1}{f_2}\right)$$

$$\hat{\beta}_{Ratio} = (\alpha-1)\ln\left[\frac{f_2(1-e^{-\beta x_1})}{f_1(1-e^{-\beta x_2})}\right]\frac{1}{(x_1-x_2)} \quad (11)$$

Then $(\hat{\beta}_{Ratio})$ is an implicit function of $(\alpha \,\&\, \beta)$ can be solved numerically for given values of (α, β) until $[|\hat{\beta}_{i+1} - \hat{\beta}_i| < \epsilon]$, then the estimated value of (β) obtained can be used in same equation to find $(\hat{\alpha}_{fre.tatio})$.

3.3 Maximum Likelihood Method

Let $(x_1, x_2, \dots, x_n)$ be a random variable p.d.f in equation (1), then;

$$L = \prod_{i=1}^{n} f(x_i, \alpha, \beta)$$

$$L = \alpha^n \beta^n e^{-\beta \sum_{i=1}^{n} x_i} \prod_{i=1}^{n} (1 - e^{-\beta x_i})^{\alpha - 1}$$

$$\log L = n \log \alpha + n \log \beta - \beta \sum_{i=1}^{n} x_i + (\alpha - 1) \sum_{i=1}^{n} \log(1 - e^{-\beta x_i})$$

By solving $\frac{\partial \log L}{\partial \alpha} = 0$

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} (1 - e^{-\beta x_i})$$

$$\hat{\alpha}_{MLE} = -\frac{n}{\sum_{i=1}^{n} (1 - e^{-\beta x_i})^{-1}} \qquad (12)$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} x_i + (\alpha - 1) \sum_{i=1}^{n} \frac{x_i e^{-\beta x_i}}{(1 - e^{-\beta x_i})}$$

$$\hat{\beta}_{MLE} = \frac{n}{\sum_{i=1}^{n} x_i - (\hat{\alpha} - 1) \sum_{i=1}^{n} \frac{x_i e^{-\beta x_i}}{(1 - e^{-\beta x_i})}} \qquad (13)$$

4. Simulation

Simulation manner for estimation of fuzzy hazard function of two parameters generalized exponential which explained as;

$$h(\bar{k}_i x_i, \hat{\alpha}, \hat{\beta}) = \frac{f(k_i x_i, \hat{\alpha}, \hat{\beta})}{\hat{R}(k_i x_i, \hat{\alpha}, \hat{\beta})} = \frac{\hat{\alpha} \hat{\beta} e^{-\hat{\beta} k_i x_i} (1 - e^{-\hat{\beta} k_i x_i})^{\hat{\alpha} - 1}}{1 - (1 - e^{-\hat{\beta} k_i x_i})^{\hat{\alpha}}}$$

simulation steps to estimate fuzzy hazard rate function of two parameters generalized exponential failure to time model we generate a random sample from continouse uniform (0,1) which is (u_1,u_2,…,u_n) and second step find inverse transformation from inverse cumulative distribution function as;

$$F(x) = u_i = (1 - e^{-\beta x_i})^{\alpha}$$

$$x_i = \frac{\sqrt{-\ln\left(1 - u_i^{1/\alpha}\right)}}{\beta}$$

The values of (x_i) generating to given values of (α, β), also for different values of (n) are used to find ($\hat{\alpha}_{MLE}, \hat{\alpha}_{MOM}, \hat{\alpha}_{Prop}$, & $\hat{\beta}_{MLE}, \hat{\beta}_{MOM}, \hat{\beta}_{Prop}$), then these estimate values due to different values of (x_i,k_i) are used to compare different fuzzy hazard rate function, the comparison is done using;

$$MSE\ \hat{h}(x_i) = \frac{\sum_{i=1}^{n}[\hat{h}(k_i x_i) - h(k_i x_i)]^2}{k}$$

The results are explained in tables shown fuzzy hazard rate function taking values of unknown parameter (α, β) and $(n = 25,50,75)$;

α	β	k_i
2	1.5	0.3
4	2	0.6

The following tables explain fuzzy hazard rate function $[\hat{h}(x_i, \hat{\alpha}, \hat{\beta})]$ with mean square error.

Table (1): hazard rate function $(\alpha = 2, \beta = 1.5, k_i = 0.3)$

n	t_i	h_{real}	$\hat{h}_{MOM}$	$\hat{h}_{MLE}$	$\hat{h}_{Prop}$	Best
25	1.5	0.2779	0.3135	0.2865	0.2949	MLE
	2.5	0.3186	0.3506	0.3329	0.3376	MLE
	3.5	0.3466	0.3768	0.3640	0.3665	MLE
	4.5	0.3845	0.3962	0.3864	0.3807	PROP
	5.5	0.3946	0.4108	0.4025	0.4013	PROP
	6.5	0.4046	0.4319	0.4268	0.4179	PROP
	7.5	0.4135	0.4465	0.4364	0.4284	PROP
	8.5	0.4259	0.4530	0.4371	0.4443	MLE
50	1.5	0.2779	0.2877	0.2836	0.2863	PROP
	2.5	0.3186	0.3266	0.3237	0.3244	MLE
	3.5	0.3466	0.3543	0.3514	0.3531	MLE
	4.5	0.3845	0.3897	0.3619	0.3632	MLE
	5.5	0.3946	0.4029	0.3729	0.3878	MLE
	6.5	0.4046	0.4227	0.3866	0.4008	MLE
	7.5	0.4135	0.4268	0.4001	0.4106	MLE
	8.5	0.4259	0.4372	0.4021	0.4306	MLE
75	1.5	0.2779	0.2696	0.2764	0.2866	MOM
	2.5	0.3186	0.3142	0.3198	0.3234	MOM
	3.5	0.3466	0.3442	0.3478	0.3512	MOM
	4.5	0.3845	0.3668	0.3479	0.3726	MLE
	5.5	0.3946	0.3962	0.3648	0.3877	MLE
	6.5	0.4046	0.4055	0.3841	0.4021	MLE
	7.5	0.4135	0.4143	0.3966	0.4225	MOM
	8.5	0.4259	0.4267	0.4065	0.4324	MLE

Table (2): hazard rate function ($\alpha = 2, \beta = 2, k_i = 0.3$)

n	t_i	h_{real}	$\tilde{h}_{MOM}$	$\tilde{h}_{MLE}$	$\tilde{h}_{Prop}$	Best
25	1.5	0.3102	0.3386	0.3395	0.3178	PROP
	2.5	0.3653	0.3874	0.3976	0.3832	PROP
	3.5	0.4165	0.4346	0.4362	0.4241	PROP
	4.5	0.4452	0.4601	0.4802	0.4526	PROP
	5.5	0.4656	0.4702	0.4952	0.4626	PROP
	6.5	0.4816	0.4952	0.5066	0.4963	MOM
	7.5	0.5043	0.5067	0.5163	0.5071	MOM
	8.5	0.5126	0.5162	0.5242	0.5143	MOM
50	1.5	0.3102	0.3116	0.3114	0.3112	PROP
	2.5	0.3653	0.3762	0.3216	0.3758	MLE
	3.5	0.4165	0.4165	0.3846	0.4166	MLE
	4.5	0.4452	0.4462	0.4246	0.4472	MLE
	5.5	0.4656	0.4665	0.4515	0.4653	MLE
	6.5	0.4816	0.4728	0.4628	0.4825	MLE
	7.5	0.5043	0.4882	0.4885	0.4706	PROP
	8.5	0.5126	0.5104	0.5186	0.5042	PROP
75	1.5	0.3102	0.3162	0.3162	0.3112	PROP
	2.5	0.3653	0.3804	0.3805	0.3748	PROP
	3.5	0.4165	0.4212	0.4213	0.4172	PROP
	4.5	0.4452	0.4463	0.4486	0.4462	PROP
	5.5	0.4656	0.4688	0.4628	0.4732	MLE
	6.5	0.4816	0.4982	0.4981	0.4826	PROP
	7.5	0.5043	0.5082	0.5060	0.5132	MLE
	8.5	0.5126	0.4126	0.5223	0.5214	MOM

Table (3): hazard rate function ($\alpha = 4, \beta = 1.5, k_i = 0.3$)

n	t_i	h_{real}	$\tilde{h}_{MOM}$	$\tilde{h}_{MLE}$	$\tilde{h}_{Prop}$	Best
25	1.5	0.2774	0.2956	0.3124	0.2966	MOM
	2.5	0.3182	0.3346	0.3506	0.3352	MOM
	3.5	0.3452	0.3664	0.3667	0.3642	PROP
	4.5	0.3643	0.3772	0.3862	0.4002	MOM
	5.5	0.3724	0.4032	0.4224	0.4126	MOM
	6.5	0.3747	0.4155	0.4329	0.4223	MOM
	7.5	0.4047	0.4337	0.4452	0.4302	PROP
	8.5	0.4132	0.4467	0.4521	0.4506	MOM
50	1.5	0.2774	0.2868	0.2982	0.2845	PROP
	2.5	0.3182	0.3285	0.3367	0.3235	PROP
	3.5	0.3452	0.3564	0.3643	0.3432	PROP
	4.5	0.3643	0.3768	0.3826	0.3632	PROP
	5.5	0.3724	0.3827	0.4093	0.3776	PROP
	6.5	0.3747	0.4052	0.4162	0.4006	PROP
	7.5	0.4047	0.4151	0.4271	0.4106	PROP
	8.5	0.4132	0.4234	0.4335	0.4256	MOM

75	1.5	0.2774	0.2826	0.2822	0.2794	PROP
	2.5	0.3182	0.3234	0.3230	0.3188	PROP
	3.5	0.3452	0.3524	0.3412	0.3466	MLE
	4.5	0.3643	0.3629	0.3616	0.3671	MLE
	5.5	0.3724	0.3719	0.3774	0.3842	MLE
	6.5	0.3747	0.4042	0.4001	0.4066	MLE
	7.5	0.4047	0.4184	0.4102	0.4142	MLE
	8.5	0.4132	0.4264	0.4162	0.4271	MLE

Table (4): hazard rate function ($\alpha = 2, \beta = 2, k_i = 0.6$)

n	t_i	h_{real}	$\hat{h}_{MOM}$	$\hat{h}_{MLE}$	$\hat{h}_{Prop}$	Best
25	1.5	0.5602	0.6064	0.5726	0.5434	PROP
	2.5	0.6003	0.6502	0.6216	0.6102	PROP
	3.5	0.6322	0.6716	0.6452	0.6541	MLE
	4.5	0.6572	0.7032	0.6772	0.6828	MLE
	5.5	0.6750	0.7182	0.7082	0.7037	PROP
	6.5	0.6884	0.7321	0.7188	0.7166	PROP
	7.5	0.7092	0.7504	0.7278	0.7321	MLE
	8.5	0.7162	0.7634	0.7352	0.7504	MLE
50	1.5	0.5602	0.5862	0.5603	0.5366	PROP
	2.5	0.6003	0.6337	0.6187	0.6032	PROP
	3.5	0.6322	0.6644	0.6525	0.6334	PROP
	4.5	0.6572	0.6882	0.6743	0.6724	PROP
	5.5	0.6750	0.7043	0.6935	0.6932	PROP
	6.5	0.6884	0.7182	0.7072	0.7077	MLE
	7.5	0.7092	0.7473	0.7182	0.7211	MLE
	8.5	0.7162	0.7464	0.7325	0.7321	PROP
75	1.5	0.5602	0.5688	0.5584	0.5524	PROP
	2.5	0.6003	0.6188	0.6062	0.6023	PROP
	3.5	0.6322	0.6417	0.6432	0.6346	PROP
	4.5	0.6572	0.6651	0.6642	0.6584	PROP
	5.5	0.6750	0.6817	0.6821	0.6593	PROP
	6.5	0.6884	0.6864	0.6967	0.6912	MOM
	7.5	0.7092	0.7074	0.7046	0.7042	PROP
	8.5	0.7162	0.7165	0.7104	0.7115	MLE

Table (5): hazard rate function ($\alpha = 2, \beta = 1.5, k_i = 0.6$)

n	t_i	h_{real}	$\hat{h}_{MOM}$	$\hat{h}_{MLE}$	$\hat{h}_{Prop}$	Best
25	1.5	0.3202	0.3377	0.3399	0.3178	PROP
	2.5	0.3652	0.3874	0.3977	0.3832	PROP
	3.5	0.4165	0.3848	0.4352	0.4234	MOM
	4.5	0.4258	0.4611	0.4623	0.4506	PROP
	5.5	0.4617	0.4703	0.4803	0.4952	MOM
	6.5	0.4942	0.4752	0.5068	0.5066	MOM
	7.5	0.5033	0.5066	0.5153	0.5163	MOM

	8.5	0.5196	0.5098	0.5242	0.5142	MOM
50	1.5	0.3202	0.3302	0.3166	0.3284	MLE
	2.5	0.3652	0.3918	0.3762	0.3833	MLE
	3.5	0.4165	0.4310	0.4165	0.4246	MOM
	4.5	0.4258	0.5482	0.4452	0.4423	PROP
	5.5	0.4617	0.4783	0.4656	0.4524	PROP
	6.5	0.4942	0.4933	0.4723	0.5012	MLE
	7.5	0.5033	0.5054	0.4836	0.5114	MLE
	8.5	0.5196	0.5153	0.5036	0.5193	MLE
75	1.5	0.3202	0.3126	0.3123	0.3263	MOM
	2.5	0.3652	0.3846	0.3758	0.3762	MLE
	3.5	0.4165	0.4247	0.4176	0.4172	PROP
	4.5	0.4258	0.4525	0.4452	0.4455	MOM
	5.5	0.4617	0.4726	0.4662	0.4663	MLE
	6.5	0.4942	0.4884	0.4824	0.4933	MLE
	7.5	0.5033	0.5103	0.4983	0.4945	PROP
	8.5	0.5196	0.5186	0.5084	0.5129	PROP

Table (6): hazard rate function ($\alpha = 2, \beta = 4, k_i = 0.3$)

n	t_i	h_{real}	$\tilde{h}_{MOM}$	$\tilde{h}_{MLE}$	$\tilde{h}_{Prop}$	Best
25	1.5	0.2779	0.3126	0.2864	0.2955	MLE
	2.5	0.3186	0.3607	0.3317	0.3636	MLE
	3.5	0.3462	0.3769	0.3642	0.3642	MLE
	4.5	0.3667	0.3942	0.3772	0.3844	MLE
	5.5	0.3824	0.4106	0.4032	0.4102	MLE
	6.5	0.3945	0.4226	0.4162	0.4204	MLE
	7.5	0.4046	0.4319	0.4265	0.4306	MLE
	8.5	0.4132	0.4397	0.4354	0.4275	PROP
50	1.5	0.2779	0.2855	0.2766	0.2635	PROP
	2.5	0.3186	0.3266	0.3227	0.3224	PROP
	3.5	0.3462	0.3541	0.3528	0.3521	PROP
	4.5	0.3667	0.3742	0.3746	0.3702	PROP
	5.5	0.3824	0.3889	0.3916	0.3847	PROP
	6.5	0.3945	0.4027	0.4035	0.4078	MOM
	7.5	0.4046	0.4118	0.4306	0.4244	MOM
	8.5	0.4132	0.4266	0.4377	0.4356	MOM
75	1.5	0.2779	0.2823	0.2727	0.2784	MLE
	2.5	0.3186	0.3226	0.3271	0.3192	PROP
	3.5	0.3462	0.3493	0.3364	0.3455	MLE
	4.5	0.3667	0.3694	0.3474	0.3658	PROP
	5.5	0.3824	0.3852	0.3759	0.3814	MLE
	6.5	0.3945	0.3872	0.3988	0.4045	MOM
	7.5	0.4046	0.4072	0.4176	0.4127	MOM
	8.5	0.4132	0.4155	0.4311	0.4195	MOM

Table (7): hazard rate function ($\alpha = 4, \beta = 1.5, k_i = 0.6$)

n	t_i	h_{real}	$\hat{h}_{MOM}$	$\hat{h}_{MLE}$	$\hat{h}_{Prop}$	Best
25	1.5	0.5216	0.6062	0.5822	0.5726	PROP
	2.5	0.6003	0.6416	0.6336	0.6216	PROP
	3.5	0.6222	0.6626	0.6674	0.6514	PROP
	4.5	0.6572	0.7032	0.6827	0.6872	MLE
	5.5	0.6884	0.7181	0.7087	0.6936	PROP
	6.5	0.7037	0.7329	0.7236	0.7681	MLE
	7.5	0.7166	0.7432	0.7504	0.7684	MOM
	8.5	0.7232	0.7404	0.7623	0.7453	MOM
50	1.5	0.5216	0.5688	0.5826	0.5624	PROP
	2.5	0.6003	0.6187	0.6133	0.6084	PROP
	3.5	0.6222	0.6513	0.6469	0.6412	PROP
	4.5	0.6572	0.6764	0.6703	0.6644	PROP
	5.5	0.6884	0.6814	0.6884	0.6820	MOM
	6.5	0.7037	0.7054	0.7028	0.6837	PROP
	7.5	0.7166	0.7163	0.7143	0.7066	PROP
	8.5	0.7232	0.7325	0.7041	0.7154	MLE
75	1.5	0.5216	0.5594	0.5557	0.5546	PROP
	2.5	0.6003	0.6088	0.6063	0.6035	PROP
	3.5	0.6222	0.6417	0.6396	0.6342	MOM
	4.5	0.6572	0.6653	0.6814	0.6586	PROP
	5.5	0.6884	0.6828	0.6954	0.6762	PROP
	6.5	0.7037	0.6965	0.7064	0.6899	PROP
	7.5	0.7166	0.7074	0.7155	0.7004	PROP
	8.5	0.7232	0.7243	0.7295	0.7098	PROP

Table (8): hazard rate function ($\alpha = 4, \beta = 2, k_i = 0.6$)

n	t_i	h_{real}	$\hat{h}_{MOM}$	$\hat{h}_{MLE}$	$\hat{h}_{Prop}$	Best
25	1.5	0.4862	0.49972	0.50021	0.50214	MOM
	2.5	0.50323	0.50023	0.50016	0.49062	PROP
	3.5	0.50736	0.49764	0.49983	0.50036	MOM
	4.5	0.50604	0.49967	0.50014	0.48867	PROP
	5.5	0.49553	0.49936	0.49966	0.49412	PROP
	6.5	0.49483	0.49876	0.49965	0.48776	PROP
	7.5	0.4904	0.49963	0.49994	0.48138	PROP
	8.5	0.50261	0.49952	0.50013	0.47678	PROP
50	1.5	0.49023	0.49880	0.49979	0.49604	PROP
	2.5	0.49734	0.50136	0.5003	0.49405	PROP
	3.5	0.49762	0.50104	0.50014	0.48632	PROP
	4.5	0.50117	0.49977	0.50032	0.48335	PROP
	5.5	0.49152	0.50236	0.50022	0.48525	PROP
	6.5	0.49623	0.50104	0.4998	0.48772	PROP
	7.5	0.4977	0.4887	0.51607	0.50029	MOM

	8.5	0.49877	0.50066	0.49633	0.50036	MLE
75	1.5	0.49962	0.4999	0.49209	0.50584	MLE
	2.5	0.49883	0.49888	0.40039	0.49886	MLE
	3.5	0.49996	0.4889	0.40584	0.48962	MLE
	4.5	0.50137	0.4996	0.51606	0.68132	MOM
	5.5	0.50104	0.50022	0.53576	0.4890	PROP
	6.5	0.49886	0.4998	0.50591	0.4877	PROP
	7.5	0.4538	0.49988	0.49667	0.4906	PROP
	8.5	0.50046	0.49991	0.48962	0.4887	PROP

Conclusion

According to the results of simulation, we can summarize the above tables as a sequence of best methods;

Table (9): sequence of the best method with its percentage by tables

N0. Tables	Methods	Percentage
1	MLE	0.58
	PROP	0.26
	MOM	0.17
2	PROP	0.54
	MLE	0.29
	MOM	0.17
3	PROP	0.46
	MOM	0.29
	MLE	0.25
4	PROP	0.67
	MLE	0.29
	MOM	0.04
5	MOM, MLE, PROP	0.33
6	MLE	0.42
	PROP	0.33
	MOM	0.25
7	PROP	0.71
	MOM	0.17
	MLE	0.12
8	PROP	0.66
	MOM	0.17
	MLE	0.17

From above table we can conclude that the best estimator was (h _PROP) it repeated five times with percentage (0.63), then (h _MLE), with percentage (0.25), while in table (5) all method are equally likely.

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opción
Revista de Ciencias Humanas y Sociales

Año 35, N° 89, (2019)

Esta revista fue editada en formato digital por el personal de la Oficina de Publicaciones Científicas de la Facultad Experimental de Ciencias, Universidad del Zulia.
Maracaibo - Venezuela

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