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#### Abstract

In present-day technology, a high level of reliability of systems is required. Recently, many authors have computed under different sets of assumptions and in varying degrees of generality the reliability function and the mean lifetime, for the renewable systems. The reliability of a renewable system, composed of n main units and one stand by, under a preventive maintenance activity. The reliability function and the mean lifetime are obtained for the cases of loaded, nonleaded (cold standby) and lightly loaded (warm standby) systems with repair and preventive maintenance. The results obtained before in [1, 4], in case of repair only, are derived from the present results as special cases.

**Keywords**: Reliability; Mean lifetime; Renewable system; Preventive maintenance; loaded standby.

# Análisis de confiabilidad para el sistema de reserva con renovables, reparación y mantenimiento preventivo

#### Resumen

En la tecnología actual, se requiere un alto nivel de confiabilidad de los sistemas. Recientemente, muchos autores han calculado bajo diferentes conjuntos de supuestos y en diversos grados de generalidad la función de confiabilidad y la vida media de los sistemas renovables. La fiabilidad de un sistema renovable, compuesto por n unidades principales y una en espera, bajo una actividad de mantenimiento preventivo. La función de confiabilidad y la vida útil promedio se obtienen para los casos de sistemas cargados, sin plomo (en espera en frío) y con poca carga (en espera en caliente) con reparación y mantenimiento preventivo. Los resultados obtenidos antes en [1, 4], solo en caso de reparación, se derivan de los resultados actuales como casos especiales.

**Palabras clave:** Fiabilidad; vida media; sistema renovable; mantenimiento preventivo; en espera cargado.

#### **1. INTRODUCTION**

The current stage of development of Ukraine is characterized by dynamic socio-economic progress, entry into the global financial system, which is reflected in social, labor and other processes. Scientific and technical development, the experience of Western countries stimulate the rapid development of various forms of employment (INSHYN, 2014a; KRASNORUTSKA, 2018). The current Code of Labor Laws of Ukraine 1971 is full of editions, amendments and does not fully comply with the modern realities of labor relations (KONSTANTINOVSKY, 2018). For the last few decades, there has been a sharp increase in new forms of employment. The reasons for this growth are multifaceted, namely:

- Increased competition as a result of globalization;

- Technological changes that contributed to the reorganization of business and labor;

- The increased participation of women in the labor market;

- The emergence of new types of contractual agreements, sometimes as a result of legal changes, as well as due to changes in business models (SCHMID, 2016).

The American Freelancers Union estimates that in early 2010, there were 42 million independent workers, and the European Commission counted 32.5 million freelancers in the EU27 (BUREAU and CORSANI, 2016). At the same time, according to Eurofound data for 2015, social and economic changes have led to the emergence of new forms of employment in Europe for both workers and selfemployed. Besides the distinction between self-employment and paid work, these new forms of employment have serious implications for working conditions and the labor market (MANDL et al., 2015).

A growing body of modern research shows that increasing labor market flexibility provides new employment opportunities for young workers, unskilled workers, minorities and immigrants. These groups of workers are represented disproportionately in the flexible employment market (BOOTH et al., 2002; KAHN, 2007). In addition, temporary employment appears to be a common way to return the unemployed to work. On the other hand, increasing the flexibility of the labor market has its consequences: since workers with flexible employment are less protected from job loss than regular workers with standard employment, they are more exposed to unemployment risks and have access to fewer jobs on average (BERTON and GARIBALDI, 2012).

From a theoretical point of view, the views on the standard of employment have changed depending on the conditions of the socioeconomic development of society. For the fast-growing service sector, those employees are required that have flexible working hours, are more mobile and, if necessary, easier to dismiss, because they have only a temporary employment contract, also, that combine executive and entrepreneurial functions, etc. (KOCH and FRITZ, 2013). Small enterprises' demand for workers has a similar character, as the role of the former in the modern economy is constantly growing (SCHMID, 2016).

The difficult socio-economic situation in Ukraine requires urgent measures to find tools to create an effective labor market in the context of other macroeconomic components of growth, which has led to increased interest in employment in the scientific community. In this regard, the theoretical and methodological foundations developed by scientists, lawyers, economists, require thorough testing in a real market environment, taking into account the achievements of foreign and domestic legal science in recent years.

The most important relative characteristic of employment is an indicator of labor activity of the population (KOUGIAS, 2017). It

gives quantitative information about the relative number of people employed in the national economy and can be calculated both for the entire employed population as a whole and for individual sex and age groups.

Employment is a form of matching labor supply in the labor market to demand for it, as well as the main indicator of labor market balance (DIMOULAS, 2014). On the one hand, employment reflects the achieved level of development, the contribution of living labor as a factor of production, and on the other hand, employment gives future proportions of production factors. Employment combines production and consumption, and its structure determines the nature of their relationship (BOERI, 2011).

Science has not developed a unified approach to understanding atypical employment. However, some scholars note that atypical employment is the labor activity of workers of a specific classification group, which is provided for and/or not prohibited by the legislation of Ukraine. At the same time, it does not fall under standard rules and requires a special legal mechanism of regulation and organizational and economic support, because of the peculiarities in the organization of the working time regime, working conditions and workplace (VAPNYARCHUK, 2016). The Organization for Economic Cooperation and Development (OECD) describes nonstandard work as all labor relations that do not meet the "norm" of regular employment with one employer over a long period of time (OECD, 2015). The International Labor Organization (ILO) believes that non-standard forms of employment consist of employment agreements that differ from "standard labor relations". As the latter are understood as fulltime work, subordinate and bilateral relations between the employee and the employer, including temporary or fixed-term employment contracts, self-employment, as well as part-time work (ILO, 2016).

Today, new forms of nonstandard work have arisen on the labor market that are not regulated by applicable labor laws, among which distance employment is worth noting. Such a phenomenon in Ukraine is in its infancy, although in many countries of the world it has spread and is in competition with conventional forms of employment (MACKENZIE and MARTÍNEZ LUCIO, 2014). In the modern globalized world, diversification of employment relations is becoming a major competitive advantage. The ability of the labor market to adapt to rapid changes in the socio-economic and institutional environment depends on such diversification (LANG et al., 2013). However, the diversification of employment relations is negatively related to the rigidity of labor laws.

For the development of atypical forms of employment in Ukraine, it is necessary to create conditions for legal support and regulation of relations of distance work, introduce state development programs and stimulate confidence in new types of work among employers and workers. Unfortunately, the legislation of Ukraine does not use the term "remote work", and the "Regulation on the work conditions of homeworkers" approved by the Decree of the USSR State Labor Committee dated September 29, 1981 No. 275/17-99 can be considered the only act regulating such relations. Thus, domestic

labor legislation lacks the categorical apparatus regarding the latest forms of employment.

The dynamic development of social processes and the world of work necessitates regular updates, and in some cases, the creation of a new regulatory framework that can meet the needs of modern society. Thus, the purpose of the article is to analyze the practice of application and features of legal regulation of the latest non-standard forms of employment in the national legislation and the legislation of foreign countries in the current conditions of socio-economic development. The study also highlights the question of international standards implementation into national legislation.

#### 2. METHODOLOGY

We confine ourselves to the case of a system composed of n main units and one spare unit under the following general conditions:

(1) As-soon as one of the main units fails, the spare one immediately takes up the load. The repair of the failed unit or the maintenance begins immediately.

(2) The repair and maintenance of a unit fully restores all its initial properties.

(3) The switch over times from any case to other are negligible.

(4) The repair times are random variables with distributions  $G_i$  (.)

(5) The inspection times are random variables with distributions  $U_i(t) = 1 - e^{-b}i^t$ , bi > 0.

(6) The maintenance times are random variables with distributions  $V_{\rm I}$  (.).

(7) The periods of failure free operation of the main units are random having the distributions

 $F_i(t) = 1 - e^{-a}i^t, ai > 0$ 

(8) The lifetime of the spare unit, in case of lightly loaded systems, is a random variables having the distribution B (t) =  $1 - e^{-b t}$ , b > 0.

(9) The system breaks down in case of two or more units failed or called for inspection simultaneously.

### **3. RESULTS and DISCUSSION**

Three models of stand by redundant systems defined as follows:

(1) The case of loaded stand by, in which the stand by unit is in the same operational state as the main units and therefore has the same intensity of breakdown.

(2) The case of nonleaded stand by (cold standby), in which the standby unit carries no load and therefore cannot fail.

(3) The case of lightly loaded standby (warm standby), in which the standby unit is loaded but not as fully as the main units and therefore has a different intensity of breakdown.

Loaded and nonleaded standby systems are special cases of the lightly loaded stand by system. Now, the Laplace transform of the

reliability function and the mean lifetime for the last three models of stand by redundant systems are obtained.

## SOME MODELS FOR THE LIFETIME DISTRIBUTION OF STANDBY REDUNDANT SYSTEMS

In this section, we investigate the reliability function and the mean lifetime for some models of standby redundant systems with repair and preventive maintenance. Denote by R (t) the reliability of failure free operation during the period (0, t).

Model 1: Loaded Standby System

In this case, the event of failure-free operation of the system during the period (0, t) is decomposable into five exclusive events:

(i) No failure and no inspection occur prior to time t with probability:

$$\prod_{i=1}^{n+1} \bar{F}_i(t) \bar{U}_i(t)$$

(ii) An inspection for the j's unit occurs at time x (x <t), the remaining units operate to time t without failure or inspection and the maintenance time of the j's unit completed after t. The probability of this event is:

$$\sum_{j=1}^{n+1} \prod_{i=1}^{n+1} \bar{F}_i(t) \bar{U}_i(t) \int_0^t \bar{F}_j(x) \bar{v}_j(t-x) dU_j(x), \ i \neq j.$$

(iii) The first inspection occurs at time x (x < t), the maintenance of this unit is completed at time y prior to time t (x< y < t), during the maintenance period the remaining units continue functioning without failure or inspection. After the completion of maintenance the system works normally to time t. The probability of this event is:

$$\sum_{j=1}^{n+1} \int_{0}^{t} \int_{\substack{0\\i \neq j}}^{t-x} \prod_{i=1}^{n+1} \overline{F}_i(x) \overline{U}_i(x) \overline{F}_j(x) \overline{F}_i(y) \overline{U}_i(y) R(t-y) - x) dv_j(y) du_j(x).$$

(iv) The j's unit failed at time x (x < t), the repair of this unit is completed after t. The remaining units operate to time t without failure or inspection. The probability of this event is:

$$\sum_{\substack{j=1\\i \neq j}}^{n+1} \prod_{i=1}^{n+1} \bar{F}_i(t) \bar{U}_i(t) \int_0^t \bar{U}_j(x) \, \bar{G}_j(t-x) \, dF_j(x).$$

(v) The first failure occurs at time x (x < t), the repair of this unit is at time y prior to time t (x < y < t), during the repair period the remaining units continue functioning without failure or inspection. From the time y to time t, the system functions normally. The probability of this event is:

$$\sum_{j=1}^{n+1} \int_{0}^{t} \int_{0}^{t-x} \prod_{i=1}^{n+1} \overline{F}_i(x) \overline{U}_i(x) \overline{U}_j(x) \overline{F}_i(y) \overline{U}_i(y) R(t-y) - x) dG_j(y) dF_j(x).$$

Therefore, the probability that the system will operate to time t without failure, where initially we start with n main units and one new loaded spare unit, is given by:

$$\begin{split} R_{1}(t) &= \prod_{i=1}^{n+1} \bar{F}_{i}(t) \bar{U}_{i}(t) + \sum_{\substack{j=1\\i \neq j}}^{n+1} \prod_{\substack{i=1\\j \neq j}}^{n+1} \bar{F}_{i}(t) \bar{U}_{i}(t) \int_{0}^{t} \bar{F}_{j}(t) .\\ \bar{V}_{j}(t-x) dU_{j}(x) + \sum_{\substack{j=1\\j \neq j}}^{n+1} \sum_{\substack{j=1\\j \neq j}}^{n+1} \bar{F}_{i}(t) \bar{U}_{i}(t) \int_{0}^{t} \bar{U}_{j}(x) \bar{G}_{j}(t-x) dF_{j}(x) +\\ \sum_{\substack{j=1\\i \neq j}}^{n+1} \int_{0}^{t} \int_{\substack{i=1\\i \neq j}}^{t-x} \prod_{\substack{i=1\\i = 1}}^{n+1} \bar{F}_{i}(x) \bar{U}_{i}(x) \bar{F}_{j}(X) \bar{F}_{i}(y) \bar{U}_{i}(y) R(t-y) \\ &- x) dV_{j}(y) dU_{j}(x) +\\ \sum_{\substack{j=1\\i \neq j}}^{n+1} \int_{0}^{t} \int_{\substack{i=1\\i \neq j}}^{t-x} \prod_{\substack{i=1\\i = 1}}^{n+1} \bar{F}_{i}(x) \bar{U}_{i}(x) \bar{U}_{j}(X) \bar{F}_{i}(y) \bar{U}_{i}(y) R(t-y) \\ &- x) dG_{j}(y) dF_{j}(x). \end{split}$$

Since,  $F_i(t) = 1 - e^{-b}i^t U_i(t) = 1 - e^{-b}i^t$ 

Then,

$$R_{1}(t) = e^{-(A=B)t} + \sum_{j=1}^{n+1} b_{j} e^{-(A+B-a_{j}-b_{j})t} \int_{0}^{t} e^{-(a_{j}+b_{j})x \overline{V_{j}}} (t-x) dx$$

$$+\sum_{j=1}^{n+1}\int_{0}^{t}\int_{0}^{t-x}b_{j}e^{-(A+B)x}e^{-(A+B-a_{j}-b_{j})y}R(t-y-x)dV_{j}(y)dx$$

$$+\sum_{j=1}^{n+1} a_j e^{-(A+B-a_j-b_j)t} \int_0^t e^{-(a_j+b_j)x} \bar{G}_j(t-x) dx$$

$$+\sum_{j=1}^{n+1} \int_0^t \int_0^{t-x} a_j e^{-(A+B)x} e^{-(A+B-a_j-b_j)y} R(t-y-x) dG_j(y) dx$$

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$$A = \sum_{j=1}^{n+1} a_j, B = \sum_{j=1}^{n+1} b_j$$

Where,

To solve this integral equation we introduce the following Laplace transforms:

$$r(s) = -\int_{0}^{\infty} e^{-st} dR(t), \qquad g(s) = \int_{0}^{\infty} e^{-st} dG(t) and$$
$$V(s) = \int_{0}^{\infty} e^{-st} dV(t).$$

Taking Laplace transform of (2.1) and using the above notations, we get:

$$r_1(s) = \frac{\sum_{j=1}^{n+1} (A+B-a_j-b_j)(A+B+S-a_j-b_j)^{-1}(a_jg_j^*+b_jv_j^*)}{(A+B+s)-\sum_{i=1}^{n+1} (a_ig_j+b_jV_j)}$$

 $g_{j} = 1 - g_{j}^{*}$  ,  $V_{j} = 1 - V_{j}^{*}$  , Where,

$$g_j = g_j \left( A + B + s + a_j - b_j \right) \quad and \quad V_j$$
$$= \quad V_j \left( A + B + s - a_j - b_j \right).$$

The mean 1 ifetime  $T_1$  of the system is defined by:

$$T_1 = -\left[\frac{dr_1(s)}{ds}\right] \qquad s = 0$$

Hence, from equation (2.2) we get:

$$T_1 = \frac{1 + \sum_{j=1}^{n+1} (A + B - a_j - b_j)^{-1} (a_j \hat{g}_j^* + b_j \hat{V}_j^*)}{\sum_{j=1}^{n+1} (a_j \hat{g}_j^* + b_j \hat{V}_j^*)}$$

where, 
$$\hat{g}_{j}^{*} = \hat{g}_{j}^{*}(A + B + a_{j} + b_{j})$$
 and  $\hat{v}_{j}^{*} = \hat{v}_{j}^{*}(A + B - a_{j} - b_{j})$ .

Model 2: Nonleaded Stand by System

In this case, the event of failure-free operation of the system during the period (0, t) 1s decomposable in to five mutually exclusive

events. Following the same analysis as in model 1 we obtain the following integral equation:

$$\begin{split} R_{2}(t) &= \sum_{i=1}^{n} \bar{F}_{i}(t) \overline{U}_{i}(t) + \\ \sum_{j=1}^{n} \sum_{\substack{i=1\\i\neq j}}^{n} \bar{F}_{i}(t) \overline{U}_{i}(t) \int_{0}^{t} \bar{F}_{j}(x) \bar{F}_{j}(t-x) \overline{U}_{j}(t-x) \bar{V}_{j}(t-x) dU_{j}(x) + \\ \sum_{j=1}^{n} \int_{0}^{t} \int_{\substack{i\neq j}}^{t-x} \sum_{i=1}^{n} \bar{F}_{i}(x) \overline{U}_{i}(x) \bar{F}_{j}(x) \bar{F}_{i}(y) \overline{U}_{i}(y) \bar{F}_{j}(y) \overline{U}_{j}(y) R(t-y) \\ &- x). dV_{j}(y) dU_{j}(x) + \\ \sum_{j=1}^{n} \sum_{\substack{i=1\\j}}^{n} \bar{F}_{i}(t) \overline{U}_{i}(t) \int_{0}^{t} \overline{U}_{j}(x) \bar{F}_{j}(t-x) \overline{U}_{j}(t-x) \bar{V}_{j}(t-x) dF_{j}(x) + \\ \sum_{j=1}^{n} \int_{0}^{t} \int_{\substack{i=1\\j}}^{t-x} \sum_{i=1}^{n} \bar{F}_{i}(x) \overline{U}_{i}(x) \overline{U}_{j}(x) \bar{F}_{i}(y) \overline{U}_{i}(y) \bar{F}_{j}(y) \overline{U}_{j}(y) R(t-y-x) \\ &- x). dG_{j}(y) dF_{j}(x) \end{split}$$

Where  $R_2$  (t) is the probability that the system will operate to time t without failure, starting with n main units and one new nonleaded spare unit.

Following the same argument as in model 1, for

$$\begin{split} & F_{i}(t) = 1 - e^{-a_{i}t}, U_{i}(t) = 1 - e^{-b_{i}t}, \\ & F_{i}(t) = 1 - r_{i}^{-1}, V_{i}(t) = 1 - e^{-b_{i}t}, \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}), \\ & F_{i}(t) = 1 - r_{i}^{-1} \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}),$$

$$T_{2} \frac{1 + (A+B)^{-1} \sum_{j=1}^{n} (a_{j} \, \hat{g}_{j}^{*} + b_{j} \tilde{V}_{j}^{*})}{\sum_{j+1}^{n} (a_{j} \, \hat{g}_{j}^{*} + b_{j} \tilde{V}_{j}^{*})}$$

$$g_{j} = 1 - g_{j}^{*} , \qquad V_{j} = 1 - V_{j}^{*} ,$$
Where,
$$g_{j} = g_{j} (A+B+S) , \qquad V_{j} = V_{j} (A+B+S) ,$$

$$\hat{g}_{j}^{*} = g_{j}^{*} (A+B) \quad and \qquad \hat{V}_{j}^{*} = V_{j}^{*} (A+B).$$

Model 3: Lightly Loaded Stand by System

In this case, the event of failure-free operation of the system during the period (0, t) 18 decomposable into nine mutually exclusive events. Following the same analysis as in model 1 we obtain the following integral equation:

$$\begin{split} R_{3}(t) &= \sum_{i=1}^{n} \bar{f}_{i}(t) \overline{U}_{i}(t) \overline{U}_{n+1}(t) \overline{B}(t) + \\ &\sum_{\substack{j=1\\i \neq}}^{n} \sum_{\substack{j=1\\j \neq}}^{n} \bar{f}_{i}(t) \overline{U}_{i}(t) \int_{0}^{t} \overline{U}_{n+1}(x) \overline{B}(x) \overline{F}_{j}(x) \overline{U}_{j}(t-x) \overline{F}_{j}(t-x) \overline{V}_{j}(t-x) \overline{V}_{j}(t-x) dU_{j}(x) + \\ &- x) dU_{j}(x) + \\ &\sum_{\substack{j=1\\i \neq}}^{n} \int_{0}^{t} \int_{\substack{j=1\\i \neq}}^{t-x} \sum_{i=1}^{n} \bar{f}_{i}(x) \overline{U}_{i}(x) \overline{B}(x) \overline{U}_{n+1}(x) \overline{f}_{i}(y) \overline{U}_{i}(y) \overline{f}_{j}(y) \overline{U}_{j}(y) R(t-y) \\ &- x) dV_{j}(y) du_{j}(x) + \\ &\sum_{\substack{j=1\\i \neq}}^{n} \sum_{j=1}^{n} \overline{f}_{i}(t) \overline{U}_{i}(t) \int_{0}^{t} \overline{U}_{j}(x) \overline{B}(x) \overline{U}_{n+1}(x) \overline{F}_{j}(t-x) \overline{U}_{j}(t-x) \overline{G}_{j}(t-x) dF_{j}(x) + \end{split}$$

$$\begin{split} \sum_{j=1}^{n} \int_{0}^{t} \int_{i=1}^{t-x} \sum_{i=1}^{n} \bar{F}_{i}(x) \bar{U}_{i}(x) \bar{B}(x) \bar{U}_{n+1}(x) \bar{U}_{j}(x) \bar{F}_{i}(y) \bar{U}_{i}(y) \bar{F}_{j}(y) \bar{U}_{j}(y) R(t) \\ &- y - x) dG_{j}(y) dF_{j}(x) + \\ \sum_{i=1}^{n} \bar{F}_{i}(t) \bar{U}_{i}(t) \int_{0}^{t} \bar{B}(x) \bar{V}_{n+1}(t-x) dU_{n+1}(x) + \\ &\int_{0}^{t} \int_{0}^{t-x} \sum_{i=1}^{n} \bar{F}_{i}(x) \bar{U}_{i}(x) \bar{F}_{i}(y) \bar{U}_{i}(y) \bar{B}(x) R(t-y-x) dV_{n+1}(y) dU_{n+1}(X) \\ &+ \\ \sum_{i=1}^{n} \bar{F}_{i}(t) \bar{U}_{i}(t) \int_{0}^{t} \bar{U}_{n+1}(x) \bar{G}_{n+1}(t-x) dB(x) + \\ &\int_{0}^{t} \int_{0}^{t-x} \sum_{i=1}^{n} \bar{F}_{i}(x) \bar{U}_{i}(x) \bar{F}_{i}(y) \bar{U}_{i}(y) \bar{U}_{n+1}(x) R(t-y \\ &- x) dG_{n+1}(y) . dB(x), \end{split}$$

Where the explanation of the first five terms is the same as that given in model 1. For the sixth term, all the n main units operate to time t without failure or inspection, but the spare unit's time of inspection comes at time x (x < t) and is completed after t. The probability of this event is:

$$\sum_{i=1}^{n} \bar{F}_{i}(t) \bar{U}_{i}(t) \int_{0}^{t} \bar{B}(x) \bar{V}_{n+1}(t-x) dU_{n+1}(x),$$

for the seventh term, all the n main units operate to time t without failure or inspection while the inspection time of the spare unit

comes at time x (x <t) and is completed at time y (x < y < t). After the completion of maintenance the system works normally to time t. The probability of this event is:

$$\int_{0}^{t} \int_{0}^{t-x} \sum_{i=1}^{n} \bar{F}_{i}(x) \bar{U}_{i}(x) \bar{F}_{i}(y) \bar{U}_{i}(y) \bar{B}(x) R(t-y) - x) dV_{n+1}(y) dU_{n+1}(x),$$

For the eighth term, all the n main units work normally to time t. The spare unit fails at time x (x < t) and its repair is completed after t. The probability of this event is:

$$\sum_{i=1}^n \overline{F}_i(t)\overline{U}_i(t) \int_0^t \overline{U}_{n+1}(x) \overline{G}_{n+1}(t-x) dB(x) ,$$

And for the last term, all the n main units work normally to time t. The spare unit fails at time x (x < t) and its repair ends at time y (x < y < t). The system works normally to time t. The probability of this event is:

$$\int_{0}^{t} \int_{0}^{t-x} \sum_{i=1}^{n} \overline{F}_{i}(x) \overline{U}_{i}(x) \overline{F}_{i}(y) \overline{U}_{i}(y) \overline{U}_{n+1}(x) R(t-y) - x) dG_{n+1}(y) dB(x).$$

Notice that  $R_3(t)$  is the probability that the system will operate to time t without failure, starting with n main units and one lightly loaded spare unit. Following the same argument as in model 1, for the same  $F_i$  (t) and  $U_i$  (t), we get that:

$$r_{3}(s) = \frac{(A+B)(A+B+s)^{-1} \left[\sum_{j=1}^{n} (b_{j}V_{j}^{*} + a_{j}g_{j}^{*}) + b_{n+1} + V_{n+1}^{*} + bg_{n+1}^{*}\right]}{s + \sum_{j=1}^{n} (a_{j}g_{j}^{*} + b_{j}V_{j}^{*}) + bg_{n+1}^{*} + b_{n+1}V_{n+1}^{*}}$$

$$\begin{split} T_{3} &= \frac{1 + (A+B)^{-1} \left[ \sum_{j=1}^{n} (a_{j} \hat{\tilde{g}}_{j}^{*} + b_{j} \hat{\tilde{V}}_{j}^{*}) + b \hat{\tilde{g}}_{n+1}^{*} + b_{n+1} \hat{\tilde{V}}_{n+1}^{*} \right]}{\sum_{j=1}^{n} (a_{j} \hat{\tilde{g}}_{j}^{*} + b_{j} \hat{\tilde{V}}_{j}^{*}) + b \hat{\tilde{g}}_{n+1}^{*} + b_{n+1} \hat{\tilde{V}}_{n+1}^{*}} \\ where , \quad g_{j} &= 1 - g_{j}^{*} , V_{j} = 1 - \hat{V}_{j}^{*} , \\ g_{j} &= g_{j} (A + B + s), \ V_{j} = V_{j} (A + B + s), \\ \hat{\tilde{g}}_{j}^{*} &= g_{j}^{*} (A + B) \qquad \text{and} \quad \ddot{\tilde{V}}_{j}^{*} = V_{j}^{*} (A + B), \end{split}$$

(I) If 
$$g_j(.) = g(.)$$
 And  $V_j(.) = V(.)$  for all  $J =$ 

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 $1.2,\ldots,\!n$  , then ,

Model 1: from equation (2.2) we get  

$$\bar{r}_{1}(s) = \frac{\sum_{j=1}^{n+1} (A+B-a_{j}-b_{j}) (A+B+S-a_{j}-b_{j})^{-1} (a_{j}g^{*}+b_{j}V^{*})}{s+\sum_{j=1}^{n+1} (a_{j}g^{*}+b_{j}V^{*})}$$

From equation (2.3) we get

$$\bar{T}_{1} = \frac{1 + \sum_{j=1}^{n+1} (A + B - a_{j} - b_{j})^{-1} (a_{j} \hat{g}^{*} b_{j} \hat{v}^{*})}{\sum_{j=1}^{n+1} (a_{j} \hat{g}^{*}_{j} + b_{j} \hat{v}^{*}_{j})}$$

Model 2: from equation (2.4) we get:

$$\bar{r_2}(s)\frac{(A+B)(A+B+S)^{-1}\sum_{j=1}^n(a_j\,g^*+b_jV^*)}{S+\sum_{j=1}^n(a_jg_j+b_jv^*)},$$

From equation (2.5) we get:

$$\bar{T}_{2} = \frac{1 + (A+B)^{-1} \sum_{j=1}^{n+1} (a_{j} \hat{g}^{*} b_{j} \hat{V}^{*})}{\sum_{j=1}^{n} (a_{j} \hat{g}^{*} + b_{j} \hat{V}^{*})}$$

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Model 3: from equation (2.6) we get:

$$\bar{r_3}(s) \frac{(A+B)(A+B+S)^{-1}\sum_{j=1}^n (a_j g^* + b_j V^*) + bg^* b_{n+1} V^*}{S + \sum_{j=1}^n (a_j \hat{g}^* + b_j V^*) + bg^* + b_{n+1} V^*}$$

From equation (2.7) we get:

$$\bar{T}_{3} = \frac{1 + (A+B)^{-1} \sum_{j=1}^{n+1} \left(a_{j} \hat{\tilde{g}}^{*} + b_{j} \hat{\tilde{V}}^{*}\right) + b \hat{\tilde{g}}^{*} + b_{n+1} \hat{\tilde{V}}^{*}}{\sum_{j=1}^{n} \left(a_{j} \hat{\tilde{g}}^{*} + b_{j} \hat{\tilde{V}}^{*}\right) + b \hat{\tilde{g}}^{*} + b_{n+1} \hat{\tilde{V}}^{*}}$$

The results in (2,4) can be obtained from the results of section (I) by putting  $V_j$  (.) =  $U_j$ (.) =0; j=1, 2,...,n.

(II) if 
$$V_j(.) = U_j(.) = 0$$
 for all  $j = 1, 2, ..., n$  then,

Model 1: from equation (2.2) we get:

$$\bar{\bar{r}}_{1}(s) = \frac{\sum_{j=1}^{n+1} (A+B-a_{j})^{-1} (A-a_{j}) a_{j} g_{j}^{*} (A+s+a_{j})}{A+S-\sum_{j=1}^{n+1} (a_{j} g_{j} (A+s-a_{j}))}$$

From equation (2.3) we get:

$$\bar{\bar{T}}_1 = \frac{1 + \sum_{j=1}^{n+1} a_j (A - a_j)^{-1} g_j^* (A - a_j)}{A \sum_{j=1}^{n+1} a_j g_j (A - a_j)}$$

Model 2: from equation (2.4) we get:

$$\bar{\bar{r}}_{2}(s) = \frac{A \sum_{j=1}^{n} a_{j} g_{j}^{*}(A+S)}{(A+S) \left[ S + \sum_{j=1}^{n} a_{j} g_{j}^{*}(A+s) \right]}$$

From equation (2.5) we get:

$$\bar{\bar{T}}_2 = \frac{1 + A^{-1} \sum_{j=1}^{n+1} a_j g_j^*(A)}{\sum_{j=1}^{n+1} a_j g_j^*(A)}$$

Model 3: from equation (2.6) we get:

$$\bar{\bar{r}}_{3}(s) = \frac{A(A+S)^{-1} \left[\sum_{j=1}^{n} a_{j}g_{j}^{*} + bg_{n+1}^{*}\right]}{S + \left[\sum_{j=1}^{n} a_{j}g_{j}^{*} + bg_{j+1}^{*}\right]}$$
$$\bar{\bar{T}}_{3} = \frac{\left[1 + A^{-1} \sum_{j=1}^{n+1} a_{j}g_{j}^{*} + bg_{n+1}^{*}\right]}{\sum_{j=1}^{n} a_{j}g_{j}^{*} + bg_{n+1}^{*}}$$

The results in (2, 4) can obtained from the results of section (11) by putting be  $g_{j}(.) = g(.)$ ; j = 1, 2,..., n.

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