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TURBINE BLADE VIBRATION LITERATURE REVIEW

*(Al Libertador Simón Bolívar,
en el bicentenario de su nacimiento)*

ABSTRACT

A comprehensive review of the world literature dealing with vibrations of turbine blades is presented. This review includes studies of single blades using discrete mass matrix methods, distributed mass studies, and finite element investigations. Analyses of blade groups by these methods are also included. The contents of 114 papers on blade vibrations are each briefly summarized with comments relating to how each publication contributes to the present state of knowledge for this subject.

RESUMEN

Se presenta una revisión de la literatura existente a nivel mundial sobre las vibraciones de álabes de turbinas. Esta revisión incluye estudios de álabes simples usando los métodos de matriz de masa, estudios de distribución de masa, e investigaciones sobre elementos finitos. Se incluye también el análisis de grupos de álabes con dichos métodos. Se resumió el contenido de 114 publicaciones sobre vibraciones de álabes con comentarios sobre el aporte de cada publicación al conocimiento de dicho problema.

EARLY TURBINE BLADE VIBRATION STUDIES

The first investigation of vibrations of turbine blades was published by Campbell (1) in 1924. Campbell discussed problems of blade failure due to tangential mode vibrations and presented guidelines for avoiding blade resonance by de-tuning. The interaction between blade natural frequencies and various flow-related excitations is conveniently summarized on the so-called Campbell diagram, which is first presented in this paper.

Early design practice and operating procedures were discussed by Allen (2) for HP, IP and LP turbine blading. Material selection, manufacturing procedures, likely causes of failure, damping, steady and non-steady steam loading, and centrifugal loading are discussed. The existing state of knowledge for stresses associated with various loadings and other procedures are reviewed. Kroon (3) applied difference calculus to find the effect of lashing wires and centrifugal force on steady stress levels and on bending deflections of turbine blade groups. An important finding was that the root bending moment increased as the lashing wire stiffness decreased, under steady steam loading. Other work done by Kroon (4) dealt with turbine blade vibrations resulting from partial admission operation. An optical system for viewing blade vibration in a rotating turbine was described. The importance of damping in turbine blades was also emphasized but only damping of blade material (elastic hysteresis) was considered in analysis.

Nolan (5) considered that steam damping was small compared with material damping but added that damping associated with friction contact surfaces (e.g. blade / disk root interfaces, sliding tie wires, tenon/shroud assemblies, etc.) was significant in decreasing vibratory amplitudes. Nolan also discussed various sources of blade excitation (variation in steam pressure, lack of uniformity in nozzles or blades, partial admission), resonant detuning of blade structures using a Campbell diagram, the need for rotating blade testing, calculation of design frequencies, and blade fatigue testing under a combination of tensile and alternating loads to determine tolerable levels of centrifugal, steady steam bending, and vibrating stress levels. The possibility of corrosion fatigue is also mentioned, and the need for blade root designs which include large fillet radii was emphasized.

Smith (6) gave an analysis of blade group natural frequencies in which specific groups of identical turbine blades were studied. The blades were rigidly fixed to the wheel at the root, and were connected to one another by a cover band at their tips. The blades were allowed to deflect due to bending but shroud extension was not permitted. Inplane tangential vibrations only were considered. No axial or torsional modes were studied. Results were presented for a 6-blade and a 20-blade group by plotting the frequency ratio vs the cover rigidity ratio, with mass ratio as a variable parameter.

Results are given for the first sideway mode, and a second group bending mode was identified. This is the first analysis of mode groups. It showed that blade group vibration is more complex than previous single blade studies had revealed.

TAPERED TWISTED SINGLE BLADE STUDIES

Modern analysis of turbine blades begins with the work of Mendelson and Gendler (7) who used"station functions" to calculate coupled bending-torsion vibrations of cantilever beams. It was assumed that the inertia and deflections are continous between specified stations along the beam. Coupled bending-torsion vibrations were calculated in this manner. The first cantilever mode was found to be 0.9 percent lower than the exact value for a uniform cantilever when one station was used, and 0.3 percent lower when two stations were used. The second mode gave results that were over 100 percent low for one station, but this error decreases to 1.8 percent lower than exact value when two stations were used.

Lo and Renbarger (8) developed equations of motion for a rotating cantilever beam. Their results agreed to within 0.1 percent with a discrete system calculation by Sutherland (9) who used Myklestad's method. Lo and Renbarger mentioned that the vibrations of a rotating beam are influenced by the orientation of the beam principal stiffness axes with respect to the plane of rotation. Vibration perpendicular to the plane of rotation causes higher natural frequencies than vibrations in the plane of rotation. Lo and Renbarger gave the following equation for the increase in natural frequency due to centrifugal stiffening :

$$\omega^2 = \omega_L^2 + \Omega^2 \sin^2 \beta$$

where

- β is the angle of blade orientation with respect to the rotor axis
- Ω is the rotational speed
- ω_L is the beam natural frequency at zero rpm
- ω is the out-of-plane natural frequency of the blade at speed Ω

Lo (10) extended the above analysis to the case of rotating beam vibrations including Coriolis acceleration effects. The direction of vibration is again not perpendicular to the plane of rotation. Other work concerning secondary inertia terms in rotating cantilevers was done by Bogdanoff (11) who developed expressions for the coupled and uncoupled torsional-bending modes. Schilhansel (12) also obtained the equations of motion for a rotating cantilever, and solved for the first natural frequency using a Ritz procedure. Agreement within 2.2 percent and 0.1 percent was obtained, compared to the exact solution for the first and second approximations respectively. Schilhansel also verified the Lo and Renbarger (8) result for the lowest natural frequency of an oriented blade.

The uncoupled vibrations of rotating beams were studied by DiPrima and Handleman (13) who developed the equations of motion in the plane of rotation, and used a variational procedure to solve these equations. These authors indicate that the results of their method showed good agreement with Rosard (14) and with the Mendelson and Gendler (7) result, but no comparison data is presented. An extension of the DiPrima paper which included coupled torsional-longitudinal motions showed that these effects were not significantly affected by pre-twist. Gere (15) and Gere and Line (16) developed equations of motion for torsional and coupled torsional -bending vibrations of thinwalled open cross-section beams. Various cross sections and end conditions were studied. Zickel (17)(18) developed the general theory of pretwist columns and beams, and studied the bending and buckling loads. Troesch, Anliker and Zielger (19) obtained equations of motion for a thin cross section beam, and solved the eighth order frequency determinant for this case. Integrals relating the vibration of uniform and tapered cantilevers were developed by Martin (20) who used these integrals to evaluate the effect of twist on turbine blade vibrations. Conway, Becker and Dubil (21) derived expressions for the transverse vibration of beams with different taper, and obtained closed form solutions in terms of Bessel functions. Clamped-free, simply supported, and clamped-clamped boundary conditions were considered, and eigenvalue results were given but without experimental confirmation. Nordgren (22) obtained an exact solution using shallow shell theory for the vibration of pre-twisted rectangular plates which showed good correlation with Reisnner and Washizu (23) and with DiPrima (13) results.

DISCRETE MASS FINITE DIFFERENCE STUDIES OF SINGLE BLADES

In 1944 Mykelstad (24) presented a method for the bending vibration analysis of rotating beams, such as propellers, turbine blades. The method is an extension of Holzer's iterative procedure for calculation of natural frequencies, in which the beam is divided into rigid discrete masses separated by massless beam sections. This method has been adapted for turbine blade analysis and is still widely used for this purpose. Myklestad's (24) discrete mass procedure for frequency calculation of beam systems is widely used by turbine designers because of its efficiency for the computer solution of blade group frequencies. However, certain qualities which influence turbine blade calculations i. e. root stiffness, shear effects, bending and torsional inertias are sometimes not easily included without empirical guidance. The Myklestad method may also be used to compute forced response of blade groups. Its effectiveness for stress calculations is inferior to that of the finite element method which can provide greater stress detail. Jarret and Warner (25) extended the Myklestad procedure for the calculation of natural frequency of a rotating, taper-twisted turbine blade. The point of root fixity was assumed as shown in figure 1. Their theoretical results and test frequency results show -2.76 percent difference for the first mode, including centrifugal stiffening. This difference is attributed to differences in root conditions during testing and in the calculation. The second and third modes of vibration showed +7.57 percent and +4.77 percent error in frequency when compared to rotating test results. The effect of bending-torsional coupling was not considered but the extension to include this effect is given in Myklestad (24).

Targoff (26) described a transfer matrix adaptation of the Myklestad procedure which allows frequency and mode shapes to be obtained directly. The complete matrix for coupled bending-torsional vibration of beam sections is also included. Sutherland (9) used the Myklestad method to study the uncoupled modes and frequencies of rotating vibrations in the plane of rotation.

In a series of papers by Plunkett (27) (28) matrix methods are used to obtain static deflection, free vibration frequencies, and forced vibration response of beams, with particular reference to turbine blades. Plunkett's work is similar to studies made by Carnegie (29), which contained analytical formulations and blade frequency data. Though not as comprehensive as Carnegie's efforts, Plunkett (30) studied rectangular and triangular cross-section plates of rectangular cantilever platform, and obtained experimentally natural frequencies and modes for many length/width ratios. These results showed good correlation with work by Barton (31) who utilized the Ritz method for plate length/width ratios ranging from 0.5 to 5.0. Using a Rayleigh-Ritz procedure Plunkett found that the method was acceptable for uniform rectangular cantilever plates without skew, but for skewed plates the frequency correlation obtained was inaccurate, compared with the Barton results. Poor results were also obtained for triangular cross sections. Plunkett suggested that finite difference procedures as described in Melosh (32) might give improved results.

In 1953 Rosard (14) obtained analytical results for twisted cantilever beams with different width to thickness ratios. A discrete - mass method was used to obtain natural frequencies for a uniform cantilever beam, for different width/thickness ratio. Good correlation is evident for straight beams, but correlation diminishes for higher twist angles. This discrepancy was attributed to the simple test procedures used.

Leckie and Lindberg (33) compared the effectiveness of four blade types of discrete analysis for blade calculations, i.e. finite difference, Myklestad, concentrated elasticity, and a combined Myklestad-concentrated elasticity approach. These procedures were applied to the uncoupled, transverse vibration of beams. The accuracy of the combined Myklestad- concentrated elasticity approach was inversely proportional to the fourth power of the number of stations or elements used. The other three methods were shown to require more elements to achieve similar accuracy in this study.

Krupka and Baumains (34) (35) used Carnegie's equations of motion and Myklestad's method to determine the uncoupled bending-bending and torsional vibrations of rotating, tapered turbine blades. Rotary inertia and shear effects were included using a lumped mass analysis. The results showed a 3.8 percent and 8.1 percent frequency difference for the first and second modes, respectively, when the shear deformation and rotary inertia effects were not included. The result is important because estimates of error values are not easy to determine. Many errors in frequency calculations may be attributed to inaccurate determination of the cross-sectional area, and the effects of shear deformation, in the blade airfoil section. Fu (36) recently used a lumped mass method to calculate the coupled bending-bending-torsional vibrations of pre-twisted rotating Timoshenko beams. The natural frequency results compared favorably with Dawson, Ghosh and Carnegie (37). A shrouded blade was also estimated by applying approximate boundary constraints to a single blade. This type of analysis is not generally acceptable because it includes other group modes.

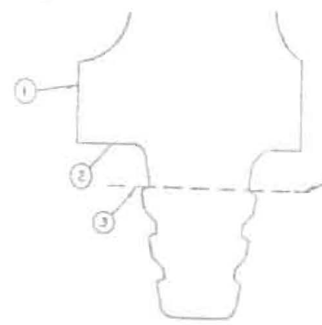
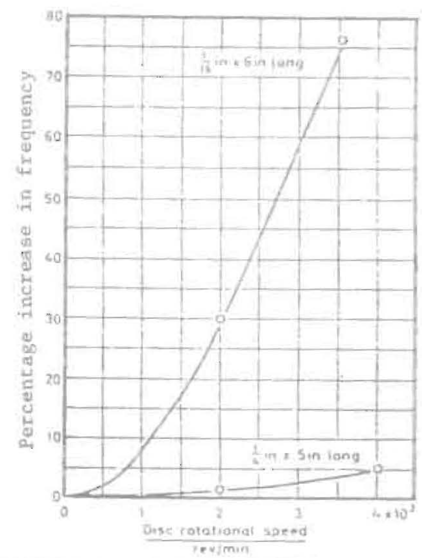


Figure 1 Assumed Point of Attachment of Beam for Analysis, From Jarret and Warner (25).



Blade	Calculation	Experiment
Zero-speed frequencies, Hz		
$\frac{1}{16}$ in × 6 in	310	299
$\frac{1}{4}$ in × 5 in	62	58

All blades 1 in wide. Disc diameter 6 in

Figure 2 Details of Frequency Rise vs. Speed for Rotating Blade. From Carnegie, Stirling and Fleming (41).

PAPERS BY CARNEGIE, THOMAS AND RAO

The static bending of twisted cantilever blades was studied by Carnegie (38) for pretwist angles between zero and 90 degrees. Deflection equations were developed using variational calculus and experimental deflection results were in good agreement with this theory (1.0%). Carnegie (39) used Rayleigh's method to solve the equations of motion for flat and twisted cantilever blades. For a straight cantilever with a uniform cross section, agreement within 0.3 percent of the well-known eigenvalue solution was achieved for the lowest bending frequency. However, experimental results were 7.3 percent lower than this theoretical result. This was attributed to differences in the modulus of elasticity and density of the blade used in the test. This point is significant. It demonstrates that the accuracy of the analytical method is only as good as material property values used.

The first four predicted torsional frequencies showed agreement to within 2 percent of measured values for pretwist angles between zero and 90 degrees. Experimental values of uncoupled vibration modes are also ahown for these angles. Experimental determination of center of flexure and center of torsion for airfoil cross sections was studied by Carnegie (40) in a discussion of bending-bending-torsion vibrations of a pretwisted cantilever blade by Hamilton's principle. Results were in close agreement with values obtained previously by Carnegie (39) using Rayleigh's method. Carnegie (29) again used Rayleigh's method to derive a frequency expression which includes the effect of centrifugal stiffening on the lowest natural frequency of a rotating cantilever. In a later paper by Carnegie, Sterling and Fleming (41) described a rotating blade test apparatus by which experimental natural frequencies were obtained for straight blades of various sizes, rotational speeds, and stagger angles (angle of vibration with respect to the plane of rotation). Most tests gave lower blade natural frequencies than the theoretical values. This is attributed to additional blade root flexibility in the test facility. For higher modes, the centrifugal increase in frequency becomes less pronounced.

Carnegie (42) (43) considered the additional effects of bending, torsion, shear deflection, rotary inertia, and Coriolis acceleration. Solutions to the equations of motion were found by variational methods and Hamilton's principle. Rao and Carnegie (44) also included the non-linear Coriolis effect in and analysis of the fundamental mode of a straight, uniform, rotating cantilever blade, vibrating in its plane of rotation. Good correlation was obtained with aralytical work by Schilhans (12) and with experiments by Carnegie, Sterling and Fleming (41).

Montoya (45) solved numerically the coupled bending-torsional equations for a single rotating, twisted cantilever blade using Runge-Kutta integration, and gave results for the first six modes of a 28 inch free standing, twisted turbine blade with a fir tree root, at various rotational speeds. Experimental data obtained from a rotating blade test showed correlation to within 1 percent for the first, second, and third modes, and to within 5 percent for the fourth, fifth and sixth modes, Data on higher modes was not given.

Rao (46) applied the Galerkin method to a Cantilever beam with a rectangular cross section and uniform taper, and found the fundamental bending frequency. The Galerkin method was shown to give consistently high results compared to the exact solution, for cases of uniform width and thickness taper. Martin's method (47) gave answers consistently lower than the exact frequency.

Carnegie and Thomas (48) used a finite difference procedure integrating the Bernoulli-Euler equation for lateral vibrations of a tapered cantilever. A solution by matrix iteration was obtained for the resulting simultaneous equations. Comparison of this method with exact results for a uniform beam showed -0.4 percent error in the first mode and less than -0.1 percent error for modes two through five. The authors show the accuracy of this method to be excellent compared with other theoretical and test results, and also with results by Rao (46) and by Martin (47).

Dawson (49) used the Rayleigh-Ritz method to determine the first five modes of coupled bending-bending vibration of pretwisted cantilever blades. Correlation with $\pm$ 2.0 percent for high aspect ratio beams (16:1) to within $\pm$ 0.4 percent for medium aspect ratio (8:1) beams, and for low aspect ratio (4:1) beams, compared to results by the transformation method of Carnegie, Dawson and Thomas (50), and with results from the matrix displacement method obtained by Dokumaci, Thomas and Carnegie (51). The mode shapes of pretwisted beams of rectangular cross section were calculated by Dawson and Carnegie (52) using a combination of matrix transfer methods and finite difference calculation. This work was extended to straight asymmetrical airfoil cross sections by Carnegie and Dawson (53). Carnegie and Thomas (54) used a finite difference procedure to calculate the bending-bending natural frequencies of tapered blades with pretwist. Frequencies and mode shapes were found for various cases of taper and pretwist. Good agreement with matrix displacement results was shown.

Petricone and Sisto (55) used the Rayleigh-Ritz method to solve the thin shell equations for natural frequencies and mode shapes of low aspect ratio, twisted compressor blades mounted on a rigid base. Results were first obtained for flat cantilever plates of rectangular and skewed geometry which compared favorably with earlier data by Barton (31). Better correlation was shown when compared against finite element results by Draper (56), whose work also included experimental data on skewed plates.

In 1970 Rao (57) published a comprehensive review of his own turbine blade frequency studies, with particular reference to use of the Ritz, Galerkin, and collocation methods. These methods are shown to be powerful analytical tools for calculation of blade natural frequencies with taper, pretwist, rotation, shear and rotary inertia, and asymmetrical cross sections. Using the Galerkin method, the effect of taper on lateral vibrations of cantilever blades was examined by Rao and Carnegie (58) who showed good correlation with experimental results from Carnegie, Dawson and Thomas

(50). The Garlekin method was further used by Rao (59) to study the effect of depth taper on torsional vibration of cantilever blades. Good correlation was shown with test results by Rao (57) for the first mode. The effects of shear deflection an rotary inertia on natural frequency were studied using the Ritz method by Rao (57). Galerkin methods for linear vibrations of blades, and for non-linear vibrations, were developed by Rao (6) (62), but without experimental confirmation. The effect of pretwist and the effect of asummetric crosssections were also studied by Rao (63) (64) using the Galerkin method. Good correlation was achieved with pretwist when compared to test results by Dawson and Carnegie (52). Good agreement was also obtained for the effect of asymmetric cross sections. Recent work by Gupta and Rao (65) using Hamilton's principle examined the torsional vibrations of pretwisted cantilever plates with aspect ratios between 1.0 to 8.0 and with pretwist angles from zero to 90 degrees. Results for zero degrees pretwist and aspect ratios from 1.0 to 4.0 compared favorably with data by Leissa (66) (0.01 percent correlation) for the first torsional mode. The Rayleigh-Ritz method was used by Swaminadham (67) to study vibration characteristics of rotating, tapered and twisted turbine blades. A rotating test rig was developed to obtain test results for rotational speeds up to 10,000 rpm. Jumaily and Faulkner (68) applied modified shell theory to investigate the vibration characteristics of hollow, symmetrical turbine blades. Correlation to within 2.0 percent was achieved with experimental natural frequency results for the first 3 modes of a single blade clamped at its base.

FINITE ELEMENT ANALYSIS

Studies which used enegy methods have shown excellent correlation with exact solutions for rectangular cross sections and uniform cantilever beams. However, turbine blades are generally skewed, tapered, twisted, airfoil sections, and exact solutions are therefore not possible. The difficulty is overcome by finite difference solution of the equation of motion, point by point throughout the structure. More recently the finite element method has been applied to turbine blade vibration analysis, first as a beam procedure and lately with a solid element approach. This is an energy method which gives accurate results when a suitable algorithm is used for eigenvalue extraction or equation solving. It is also well adapted to the digital computer because of its matrix formulation, and is simple to use because the element stiffness, mass and damping matricies may be formed internally. A rigorous basis for the finite element method may be found in the following papers for turbine blades, and in Zienkiewicz (64).

In 1961, Melosh (32) developed an element stiffness matrix for a simple plate and formed a structural matrix from assembly of plate stiffness matricies. Dawe (70) developed a dynamic matrix (stiffness and inertia) for a 12 x 12 quadrilateral plate, with one displacement and two rotational degrees of freedom at each node. Using this matrix, the natural frequencies for a uniform cantilever plate with a 3 x 3 element grid were computed, and were compared to exact results given by Barton (31). Correlation for the first five natural frequencies was within 1.4 percent, 2.8 percent, 4.3 percent, 1.1 percent, and 4.8 percent error respectively. When compared to the test results the finite element method gave 1.2 percent, 2.5 percent, 5.5 percent, -1.1 percent, and 3.5 percent error for the same cases. Though good correlation is shown, the power of the finite element method is not demonstrated in this case because a uniform cantilever plate was calculated. The method also handles tapered twisted structures with the same convenience.

Dokumaci, Thomas and Carnegie (51) gave 8 x 8 mass and stiffness matricies for a twisted beam. They then formed the structural dynamic matrix for a twisted cantilever beam for bending vibrations with 2 displacement and 2 rotational degrees of freedom per node. Frequency results for 3 and 4 element cantilever models give less than 0.1 percent difference, for the first 4 freqqencies, with 30 degrees and 90 degrees pretwist, when compared with analytical work by Anliker and Troesch (71). This method was extended to include tapered and twisted blades by Thomas, Dokumaci and Carnegie (72), whose results showed less than 1.0 percent difference for the first 5 frequencies at 30 degrees, 60 degrees and 90 degrees pretwist, when compared with finite difference results by Carnegie, Dawson and Thomas (50).

Rawtani and Dokainish (73) studied the static deflections of a pretwisted cantilever plate. A triangular shell element having a 9 x 9 bending stiffness matrix and a 6 x 6 in plane stiffness matrix was developed. An 18 x 18 element stiffness matrix was formed, by incorporating local stiffness matricies into a global stiffness matrix. Deflections of cantilever plates involving 2x2, 3x3, 4x4, 5x5, and 6x6 element grids for a 6 in. x 6 in. x 0.125 in. cantilever were examined, with pretwists of 0 degrees, 20 degrees, 40 degrees, and 60 degrees. Convergence based on gridwork were compared to results by Chetty and Tottenhan (74), who used Bongard's shallow shell equations. Tip deflections agreed to within 3.6 percent.

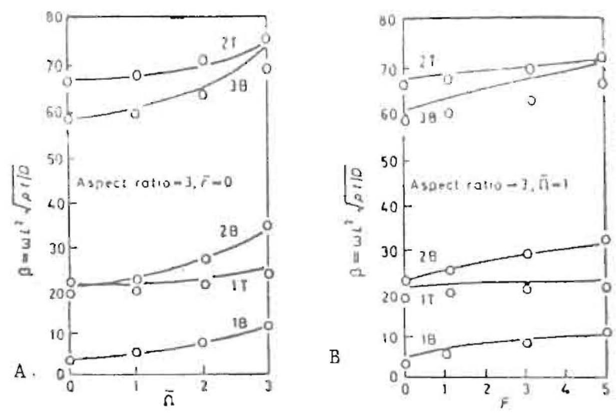
Rawtani and Dokainish (75) applied this triangular element to the vibrations of pretwisted cantilever plates. Cases of 0 degrees, 40 degrees and 80 degrees pretwist were calculated with 3x3, 4x4 and 5x5 meshes. A 5x5 mesh for a square cantilever plate showed frequency agreement to better than 2.0 percent for the first modes when compared to Barton's results. A further comparison with experimental results by Carnegie (39) for seven different pretwist angles from 0 degrees to 90 degrees showed correlation to within 5.0 percent for the first three bending modes.

Dokainish and Rawtani (76) extended their triangular plate element formulation to calculate the natural frequencies of rotating, flat cantilever plates. Centrifugal stiffening was included as a two pass operation by calculating the in-plane stress distribution under rotational force and then using these stresses to increase the plate bending stiffness in a manner similar to plate stability analysis. Comparison of 4x4 and 5x5 patterns gave a maximun variation of less than 1.6 percent, for the first five modes. Centrifugal stiffening coefficients were calculated and tabulated for the first three bending modes and the first two torsional modes, for aspect ratios of 1, 2 and 3. Rawtani and Dokainish (77) further developed an 18x18 triangular shell element with three displacement degrees-of-freedom and two in-plane rotational degrees of freedom. The preceding centrifugal stiffening meth- of was again used to obtain the natural frequencies of low aspect ratio, twisted turbomachinery blades. The effect of disk radius, pretwist and speed of rotation were studied for different blade angles. Dokainish and Rawtani (78) also discussed the "pseudo-static" deformation caused by centrifugal force and free vibration. The same 18x18 triangular shell element was again used by Rawtani (79) to study the effect of camber on the natural frequency of low aspect ratio blades. Results for 2x2, 3x3, 4x4 and 5x5 grid patterns were studied, for camber angles between zero and 90 degrees and aspect ratios of 1.0, 2.0 and 3.0. Experimental results for a 30 degree camber blade agreed within 6.58 percent for the first four modes, and with 3.93 percent and 2.79 percent agreement for the first two modes.

Rawtani (80) introduced variable thickness to the 18x18 triangular element and calculated the natural frequencies of wide chord low aspect ratio compressor blades. A 7x3 thickness grid pattern was modeled, with four angles of pretwist from zero to 34.0 degrees. Agreement with Plunkett (30) within 5.0 percent for the first two modes, for all pretwisted was achieved. An actual compressor blade was modeled by 5x5 mesh, and natural frequencies for the first two modes agreed with experimental data by 3.35 percent and 2.72 percent respectively. Dokainish and Gossain (81) used the constant thickness shell element mentioned previously to calculate the natural frequencies of an automobile fan blade at different rotational speeds.

Studies of cantilever plate vibrations were undertaken by Anderson, Irons and Zienkiewicz (82) using a triangular plate element developed by Zienkiewicz and Cheung (83). For the first three modes of vibration, a four-element model gave results within 4.0 percent of test results by Plunkett. Similar agreement was achieved with results by Barton (31). Subsequently a 64 element model gave results with 2.0 percent of test data by Plunkett (30) for the first 12 modes. Results for a 8 element skewed cantilever also showed excellent correlation with Barton's (31) results.

Ahmad, Anderson and Zienkiewicz (84) used a thick shell isoparametic solid threedimentional element to model a single tapered, twisted gas turbine blade. Natural frequencies were obtained and compared to thin shell analysis, and to test results from a unspecified source. The thin shell model gave +23.6 percent, +46.2 percent and -3.5 percent discrepancies with blade frequencies test



A

Variation of natural frequencies with speed of rotation.

D = Plate rigidity.
L = Length of plate.
r = Radius of disc.
$\bar{r}$ = r/L.
t = Thickness.
β = Non-dimensional frequency of vibration.
ρ = Density.
Ω = Speed of rotation.

B

Variation of natural frequencies with disc radius.

$\bar{\Omega}$ = Ω/ω_o = non-dimensional speed of rotation
ω = Natural frequency of rotating plate.
ω_L = Natural frequency at zero speed of rotation.
1B = First bending mode.
2B = Second bending mode.
3B = Third bending mode.
1T = First torsional mode.
2T = Second torsional mode.

Figure 3 Details of Frequency Rise vs. Speed for Rotating Plates From Dokainish and Rawtani (76).

results for the tangential, axial and torsional modes respectively. The thick shell model gave results which agreed by +0.2 percent, +27.6 percent and -6.9 percent with the test results. The thick shell agreement was generally better probably because shear effects are included. The first four mode shapes are also shown. Bossak and Zienkiewicz (85) added centrifugal stiffening to the thick shell element and achieved good correlation with blade vibrations results by Carnegie, Sterling, and Fleming (41) and by Dokainish and Rawtani (76). These results are shown in figures 2 and 3.

Olson and Lindberg (86) developed a four node cylindrical shell element with seven degrees of freedom per node for analysis of a curved fan blade. A 4x4 element grip gave results for the first 12 modes that agreed within 10.0 percent with experimental results, with the poorest correlation in the lower modes due to rigid body effects. Good agreement was obtained between the calculated mode shapes and the experimental mode shapes. Lindberg, Olson and Sarazin (87) developed a high-order triangular shell element with a 36x36 stiffness matrix, and made extensive studies on uniform-and tapered thickness fan blades, and with a clamped cylindrical shell panel. Excellent correlation with test results was achieved in each case. Rieger and Walker (88) developed a 40 degree of freedom element with parabolic thickness taper and calculated natural frequencies of a curved fan blade. Comparison of natural frequency results with Olson and Lindberg (86) experimental data, as well as with the Olson-Lindberg 28x28 rectangular element was made. With exception of the 4th mode, the first 6 modes were all within 5.5 percent of experimental results.

Other finite element studies associated with turbine blades were made by Hofmeister and Evenson (89). Modified eight and twelve node solid element with 3 degrees of freedom per node were used to model a turbine blade and other items. Turbine blade natural frequencies for a 14 element model showed better than 10.0 percent correlation for the first 6 modes compared to experimental results by Aprahamian, Overye, Evenson and Hofmeister (90). Inaccurate representation of root clamping was given as the reason for the analytical frequencies being higher for the first five modes. A rigid foundation was assumed. A parabolic shell element was used by Thomas and Mota Soares (91) to calculate the natural frequencies of a twisted blade. For setting angles (angle of the base with respect to plane of rotation) of 0 degrees, 30 degrees, 60 degrees and 90 degrees, agreement to 2.6 percent was achieved for the first mode in comparison to results by Dokainish and Rawtani (75). The effects of disk radius, angle of pretwist, angular velocity and thickness of the rotating blade were also examined.

Henry and Lalanne (92) studied the natural frequencies of rotating compressor blade using a plane, triangular, variable thickness shell element with six degrees of freedom per node. A 160 element model of a compressor blade and root-section was used to obtain frequency data at zero, 5,000 and 10,000 rpm. Correlation of the natural frequencies at zero rpm with model test frequencies was within 10.0 percent for the first 10 modes. Trompette and Lalanne (93) used nine 24 node, three d.o.f./node isoparametric thick shell elements to model a gas turbine blade for natural frequency and mode analysis. The first three zero-rpm modes agreed with test results to within 0.2 percent, 6.5 percent, and 2.0 respectively. Temperature effects on Young's modulus were included, and root flexibility was modeled by corner springs. An extension of this work by Lalanne, Henry and Trompette (94) used triangular shell elements to obtain steady centrifugal stress data for a radial inflow turbine blade. This is one of the first publications which includes any type of stress results. Natural frequencies using the 24-node thick shell element were obtained. Correlation was obtained to within 11.0 percent of test frequencies at zero rpm, for the first three modes of a four element model. Also, centrifugal stresses and natural frequencies of a gas turbine blade were calculated using triangular shell elements and 24-node brick elements, respectively. Results showed similar correlation to the preceding results.

The general purpose finite element program NASTRAM was used by MacBain (95) to study natural frequencies and mode shapes of cantilever plates with varying twist and centrifugal loading. A quadrilateral plate element called CQUAD2 was used. An 11x24 element mesh was used which gave -1.2, -2.1, 1.0 and -2.4 percent error with respect to Barton (31). The first 10 natural frequencies were calculated for 0, 12, 17, 23.5, 30 and 28 degrees of pretwist. These results agreed to within 10.0 percent with other experimental work done by MacBain. The first ten normalized mode shapes showed excellent correlation with holographic mode shapes by Mc Bain, figure 4.

Allen and Erickson (96) recently used NASTRAN to analyze a free standing gas turbine blade, and compared their results with rotating test natural frequencies. Steady-state deflections and stresses due to centrifugal force, were calculated, together with the first four mode shapes and relative vibra-

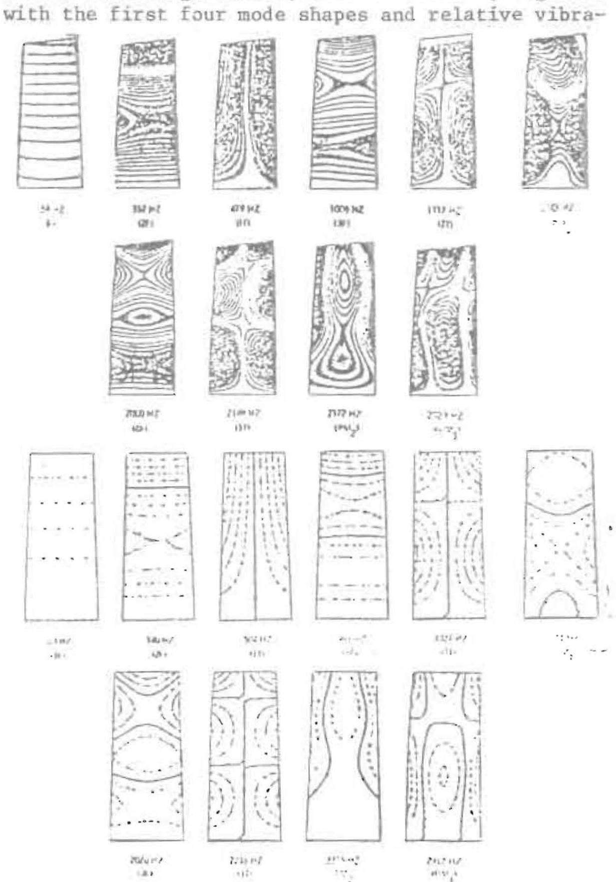


Figure 4 Comparison of Holographic and Calculated Mode Shapes For Fan Blade, From MacBain (95).

tory stress patterns. Again, no non-steady excitation data was available and no relative stress levels were included.

Barton, Scheurenbrand and Scheer (97) used flat triangular elements with five d.o.f./node (no in-plane rotation), similar to Dokainish, to obtain both natural frequencies and static and dynamic stress distributions for turbopump inducer blades. Poor correlation was obtained for the cantilever plate frequencies when results were compared to the exact values by Barton (31). This was attributed to excessive element aspect ratio values in the modeling. Dynamic stress levels are also given, but these are only normalized values because the exciting forces were unknown. Though this analysis could not be used to predict fatigue failure, it is still the closest procedure to a complete fatigue analysis found in the rotating turbomachinery blading literature.

BLADE GROUP ANALYSIS

Most of the work published on turbine blade stress and vibration problems deals with analytical and experimental procedures for single blade calculations. Though many "free standing" blades exist (mostly in gas turbines), almost all blading in steam turbines consists of blade groups which are joined together with a tip shroud band and/or with mid-span lashing wires. These mechanical couplings must be included in the dynamic analysis of steam turbine blades.

In 1956 Prohl (98) developed a discrete-mass procedure based on Myklestad's method to calculate the natural frequencies and dynamic stresses of a group of identical uniform blades joined by a cover at their tips. The principal axes of the blade cross sections were assumed to be parallel and perpendicular to the plane of the disk, and the center of twist of the blade cross section was assumed to coincide with the center of gravity. Because of the in-plane symmetry of the blade group, modes occur in the plane of the disk, and perpendicular to it. This fact was used by Prohl to uncouple the modes in his analysis, thereby simplifying the calculation of resonant frequencies and dynamic stresses.

Resonant stresses are calculated by the following formula:

$$\sigma_v = K \frac{\pi}{\delta} S \sigma_s$$

where

- σ_v is the dynamic stress at the blade root due to forced excitation
- K is the resonant-response factor
- δ is the logarithmic decrement of damping
- $K\frac{\pi}{\delta}$ is the amplification factor
- S is the fractional value of the vibration stimulus
- σ_s is the blade root steady due to the steady steam force

A sample natural frequency and resonant stress calculation is given by Weaver and Prohl (99), who showed charts of the variation of resonant response factor K, versus the harmonic order (per rev) excitation n, times the number of nozzles K, divided by the number of blades, m. The ratio, nK/m, is shown to be a significant design factor because it may be used to minimize the response factor, However, the fractional value of the stimulus, S, is not an easy number to obtain because in general blade excitation data is not readily available. Prohl's method applied for blade groups in which the tangential and axial motions may be uncoupled. This eliminates the majority of low-pressure blade groups, which experience significant modal coupling due to axial twist along the blade lenght. Also, only non-rotating natural frequencies may be calculated by this method as presented, though the extension to include rotational centrifugal effects is readily incorporated.

To account for coupled bending-bending vibration of blade groups, and for torsional coupling induced by grouping the blades, Deak and Baird (100) developed a discrete mass procedure to calculate natural frequencies of rotating tapered twisted turbine blade groups. Assuming that the center of flexure coincides with the center of gravity, two differential equations of motion for bending and one for torsion are obtained. The resulting finite difference expressions were solved by the Myklestad recurrence procedures. Cover and tie-wires are treated as massless beam sections, and the tie-wire mass is lumped at the corresponding blade locations.

A solution procedure similar to that given by Weaver and Prohl is used to solve the frequency determinant for the natural frequencies of the group. Figure 5 shows the relative mode shapes of a four blade group with two-tie wires. A Campbell diagram from Deak and Baird for this case is shown in figure 6. Calculated natural frequency results are compared with test results for the same group, at speeds from zero rpm to 3600 rpm. Agreement to within five percent for four modes was obtained.

Tuncel, Bueckner and Koplik (10) used the method of 'Diakoptics' to establish the discrete mass equations for a blade group structure. This method presumes the existence of weak coupling between blades, tie-wires, and cover. In general, results obtained for groups with longer, more flexible blades were in better agreement with test results than results obtained for short, more rigidly coupled blades.

Fleeting and Coats made a comprehensive analysis of certain blade failure which occurred in the "Queen Elizabeth II" turbines. A full description of the failed 9th and 10th stages of the high pressure rotor was given. It was concluded that operation close to the first tangential group resonance caused high vibratory stresses. These stresses when combined with centrifugal and steady steam stress,

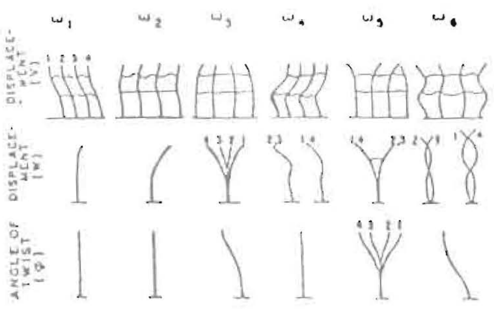


Figure 5 Relative Mode Shapes of a Four-Blade doubly-Laced Blade Group at 3600 rpm. From Deak and Baird, (100).

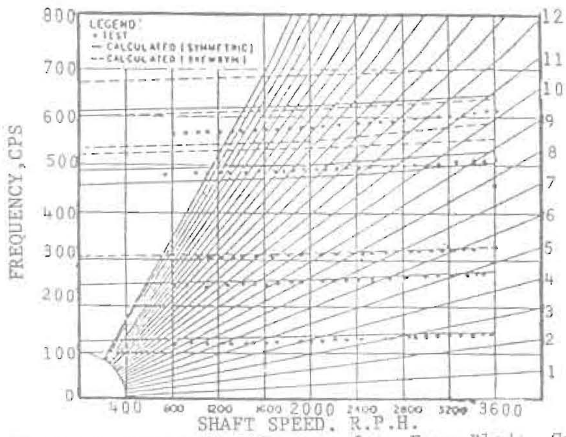


Figure 6 Campbell Diagram for Four-Blade Group eith Two Tie-Wires, From Deak and Baird (100).

caused fatigue to occur. A stress concentration at the trailing edge of the blade section at the platform was the crack initiation point.

Provenzale and Skok (103) developed a procedure for calculating natural frequencies of turbine blade groups. A procedure for calculating blade group vibrations is given which includes effects attributed to 'component response phasing' which, it is claimed, may contribute significantly to the resonance problem. This aspect is included in the Campbell diagram of the sample calculation. The conclusions of this paper are controversial and in this writer's opinion, the proposed mechanism cannot occur in an integral structure. A review of this paper has been given by Sohre (104).

Salama and Petyt (105) used simple beam finite elements to study the free tangential vibrations of a six blade group. Elements consisting of linear and cubic displacement functions were used for longitudinal and transverse vibration. Cases of harmonic excitation and partial admission are mentioned, and the respective influences of blade stiffness ratio, blade mass ratio, number of blades in a group, size and position of tie-wire, speed of rotation, and other effects are studied. Rigid root stiffness was considered, and no comparison with experimental results are included.

No numerical details of harmonic excitation or partial admission forces are given, but some blade response results under impulse loading are included. In practice, solid element formulations are now being used because they give better accuracy and stress detail. Also, the entire 3D geometry of the blade group and disk root region may be considered with solid elements.

Rao (106) used Hamilton's principle to establish the equations of motion for tangential vibration of blade groups. His closed form solution gave the first 5 natural frequencies with less than 0.25 percent difference from an analysis using Prohl's method. Rigid root stiffness is assumed and no test results are given. The method is efficient because of the refined deflection functions. Only simple geometries are considered. Eight node isoparametric solid elements were used by Sagendorph (107) to model a fan blade with a tie-wire. Calculations were made for free and for a locked shroud configurations. Comparison of the free shroud condition with experimental data gave agreement to within 10.0 percent for the first ten modes. Holography was used to verify mode shapes, which compared favorably with the finite element mode results. The locked shroud condition is not a true blade group configuration, but only a boundary condition input on a single blade. Similar work by Hall and Armstrong (108) for interlocking shrouds used the finite difference method. Srinivasan, Lionberger and Brown (109) used the component mode technique to obtain blade group natural frequencies, where the finite element method was used to obtain the motions of a single blade clamped at the root, but influenced by adjoining blades at the interlocking shroud location. This raises questions concerning the dynamic interaction of adjoining blades, to which the authors refer as 'group' modes. Damping from the rubbing of the interlocking shroud is considered at the shroud constraints. Results of the component mode method were compared with NASTRAN results for cyclic symmetry calculations, but no experimental confirmation is included.

Rieger and Nowak (110) analyzed fatigue stresses in steam turbine blade groups using the finite element method. Steady stress levels, natural frequencies and mode shapes, and dynamic stress levels in blade groups were discussed, and sample results for a given blade group are presented. A Goodman diagram procedure is used for fatigue evaluation. Non-steady excitation values were obtained from model tests with the hydraulic analogy by Rieger and Wicks (111). This appears to be the first comprehensive method in the open literature giving a full fatigue analysis procedure for turbine blade groups. Experimental verification for frequency results is obtained, but no verification is available for the fatigue results. Rieger (112) has discussed the finite element analysis of turbomachinery blade problems in a brief but comprehensive review of work in this area.

A detailed discussion of the state-of-the-art for blade vibration studies has been given by Nowak (113).