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THE ELUSIVE UNIFIED FIELD THEORY?

*(Al Libertador Simón Bolívar,
en el bicentenario de su nacimiento)*

ABSTRACT

The proposal that the Einstein field equations ($R_{\mu\sigma} = 0$) originally conceived solely for the gravitational field in a matter free region of space, actually constitute a possible set of unified field equations is re-examined. The thought process involved in arriving at this conclusion is shown to imply a "multiplicative' approach to the description of matter. Line elements for free space and for an isotropic medium both in a spherically symmetric gravitational field, that are wavelength and frequency dependent with respect to electromagnetic radiation, are introduced and their significance is discussed.

RESUMEN

Se examina de nuevo la proposición de que las ecuaciones de campo de Einstein ($R_{\mu\sigma} = 0$) constituyen un conjunto de ecuaciones de un campo unificado aunque estas ecuaciones fueron concebidas solamente para el campo gravitatorio en el espacio, libre de la materia. Se demuestra que el razonamiento por el cual se llega a esta conclusión implica un concepto 'multiplicativo' para la representación de la materia. Se introducen elementos métricos para el espacio libre y un medio isotrópico en la presencia de un campo gravitatorio de simetría esférica, los cuales dependen de la longitud y de la frecuencia de la radiación electromagnética. Se trata la importancia de estos elementos métricos.

INTRODUCTION

The special theory of relativity had its origin in the development of electrodynamics while the general theory has developed out of gravitation (1). The two theories are still basically subjected to this separate treatment and have defied even the thirty year effort of Einstein at unifying the two concepts. Einstein once stated (2) "... the idea that there exist two structures of space independent of each other, the metric gravitational and the electromagnetic, was intolerable to the theoretical spirit". This desire to incorporate electromagnetic theory and general relativity into a unified field theory is still very much alive today. C. N. Yang (3) has recently outlined some of the efforts that have been made over the years in this direction. In a recent article (4) it has been proposed that the Einstein field equations ($R_{\mu\sigma} = 0$) constitute a possible set of unified field equations. In this paper an attempt is made to present the process of reasoning and of re-examination of certain relativistic concepts that have led to this conclusion. Suggestions for the further development of these ideas are also made.

BASIC CONCEPTS OF RELATIVITY

The first concept that must be examined is that of the constancy of the velocity of light. When Einstein proposed this idea in developing the special theory of relativity he stated (1) "... that light is always propagated in *empty* space with a definite velocity 'c' which is independent of the state of motion of the emitting body". It is very clear then that this proposal refers only to empty space and not to a space containing matter. To further understand this proposal it is necessary to consider another paper in which Einstein discussed the influence of gravitation (thus involving the general theory) on the propagation of light where he stated (1) "... that the *velocity* of light in the *gravitational* field is a *function* of the *place* ...". In order to reconcile these two statements it must be concluded that the special theory of relativity can only be considered as a limiting case. When applied to experiments on the earth's surface it is only a first approximation since there are two conditions which are not met viz. i) the presence of an atmosphere (space with matter) and ii) the presence of the earth's gravitational field. Thus , the special theory of relativity is never strictly applicable since there is no way in which the gravitational field can be completely eliminated as there will always be the observer and his laboratory whose finite mass will always result in such a field even though it may be very small. Stated otherwise, there is no known practical way to obtain a perfect inertial reference frame.

The Minkowski line element (1,5) :

$$ds^2 = c^2dt^2 - (dr^2 + r^2\ (d\theta^2 + \sin^2\theta d\phi^2)) \qquad (1)$$

(given here in spherical polar co-ordinates) may be used as the starting point for obtaining the transformations due to uniform relative motion that are characteristic of special relativity. In the development of the general theory of relativity a set of tensor field equations were proposed whose solutions would show the gravitational effect (6) of matter in a matter free region of space (ie. take account of the second condition not included in special relativity). This is based on the contracted Riemann tensor or Ricci tensor and results in the Einstein free space tensor field equations ($R_{\mu\sigma} = 0$). The best known exact solution of these field equations is that due to Schwarzschild (6,7) :

$$ds^2 = (1 - (2m/r))c^2dt^2 - (dr^2/(1 - (2m/r)) + r^2 (d\theta^2 + \sin^2\theta\ d\phi^2)) \qquad (2)$$

for a spherically symmetric gravitational mass where m is the geometric mass. It should be noted that the Minkowski line element is also a solution (6) of these field equations ($R_{\mu\sigma} = 0$). If these two solutions are compared one cannot help but notice that the effect of the gravitational field is to modify the metric coefficients of the terms dt^2 and dr^2 in the line element. This consequently causes the co-ordinate velocity of light in the r direction to be different from that in the θ and ϕ directions, an effect expected from the symmetry of the problem.

In the further development of general relativity, the basic difficulty is to treat regions of space which contain matter (ie. take account of the first condition not included in special relativity) Einstein's approach was to define another set of field equations making use of the Einstein tensor ($G_{\mu\sigma}$) and the energy momentum tensor ($T_{\mu\sigma}$) in an attempt to include the effect of matter (1,6). This approach has not however produced the desired result (3). Einstein has observed concerning this procedure (8) that "The present relativistic theory of gravitation is based on a separation of the concepts of 'gravitational field' and of 'matter'. It may be plausible that the theory is for this reason inadequate for very high density of matter. It may well be the case that for a unified theory there would arise no singularity". This is an interesting observation because it can be argued that by defining another set of tensor field equations one is right away treating the two concepts differently. This suggests then that the very same field equations ($R_{\mu\sigma} = 0$) should also describe regions of space containing matter. The question is how to proceed in such a quest ?

OUR NOTIONS OF MATTER

There is another aspect of this problem which requires discussion since it helps to orient such a search. This concerns our concept of the nature of matter. The ancient Greeks were the ones who originated the discussion as to whether matter was continuous or discontinuous. Democritus proposed a discontinuous approach (9,10) in terms of atoms in the 5th. century B.C. Later the classical atomic theory of matter, again a discontinuous approach (though somewhat different (9) from that of Democritus) was re-formulated by Dalton in 1802. Here again division could proceed up to the atom (though there were different ones for the different chemical elements), a particle that was supposed to be indivisible and indestructible. At the end of the last century however, certain properties of matter were discovered which could not be accommodated in that theory and which led to the proposal (or discovery) of the electron - a charged particle smaller than the atom This in turn led to the need for the nucleus and then in turn to the proton and the neutron. This process of proliferation of sub-atomic (or 'elementary') particles has continued. Today there are over one hundred ! It is well to ask ourselves what this proliferation means? What exactly is an 'elementary' particle? In a recent article (11) H. Georgi and S. L. Glashow stated "The notion of what are the 'elementary' of structureless constituents of matter keeps changing as we are able to probe smaller and smaller distances with higher and higher energies. As long as we were limited by the energy available in chemical processes, the elementary particles were atoms; later they were protons, neutrons and electrons; currently we can smash matter into pieces sufficiently fine that quarks and leptons *appear* to be the elementary constituents of matter". It is my personal opinion that with the division of the atom and hence the violation of the atomic theory of matter one is faced with two possibilities :

a) Look for a smaller particle or particles that are indestructible and indivisible (or as Georgi and Sheldon say 'structureless').

b) Give up on the concept that matter is discontinuous!

So far science has proceeded according to the first alternative with the resultant effect that more and more sub-atomic particles are discovered.
The second approach it should be emphasized, does not negate either the existence or any possible practical importance of these 'sub-atomic' particles. It merely does not attach any fundamental significance to them.

This dilemma is at the heart of the present problem since the atomic theory or matter considers an 'additive' approach (12) to a description of matter as more fundamental than a 'multiplicative' approach. In the 'additive' approach matter is built up of 'microscopic' entities (whether they be atoms, electrons, protons, neutrons, quarks, leptons or whatever). The 'multiplicative' approach concerns itself only with the 'macroscopic' observable. It is interesting to point out that this second approach is much more suited to the theory of relativity since it is more amenable to a field treatment (5). In a unified field theory these disparate concepts must somehow be reconciled.

A MULTIPLICATIVE APPROACH

The previous discussion suggests that the search ought to be concentrated on the macroscopic properties of matter. There is one such property which has occupied my thoughts for the past ten years. This is the refractive index of a material medium defined classically as the ratio of the velocity of light in free space to that in the medium. Moreover there was a particular attempt by two researchers, E. Bortolotti (12, 13) and R. K. Luneberg (12, 14), to describe the motion of light rays along geodesics in such a material medium using a non-Euclidean space with the following line element :

$$ds' = nds = n(dx^2 + dy^2 + dz^2)^{1/2} \tag{3}$$

where n is the refractive index. The treatment is essentially non-relativistic since the time co-ordinate is not included. However the basic idea is there, which is to try to include the effect of the material medium in the metric coefficients. The only problem is how to adapt this idea to a relativistic treatment. To do this it was necessary to go back to the classical formulation of the refractive index (n_c) based on the properties of wave phenomena (15) :

$$n_c = c/v = c/\lambda_1\nu_1 = \lambda\nu/\lambda_1\nu_1 = \lambda/\lambda_1 \tag{4}$$

where c is the velocity of the electromagnetic radiation (henceforth EMR) in free space and v is its velocity in the medium; λ is the wavelength and ν is the corresponding frequency of the EMR in free space. The subscript '1' indicates the value for the respective quantities in the medium. The final equality is obtained by making the usual classical approximation that $\nu = \nu_1$. To obtain more general expressions however, this approximation is no longer made at the outset.

By comparison with the Minkowski line element for free space two possible relativistic type line elements (15) for an isotropic medium that would satisfy the classical formulation of the refractive index (equation 4)) are :

$$ds^2 = (\lambda_1^2\nu_1^2/c^2)c^2dt^2 - (dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \tag{5}$$

and :

$$ds^2 = c^2dt^2 - (c^2/\lambda_1^2\nu_1^2)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2) \tag{6}$$

Equation (6) is analogous to the Bortolotti and Luneberg line element. It is interesting to note that both these line elements (equations 5) and 6)) satisfy the Einstein field equations ($R_{\mu\sigma} = 0$). There is however a third line element (15) which also satisfies these field equations while also maintaining the co-ordinate velocity of EMR independent of direction and position - a necessary condition for an isotropic material medium :

$$ds^2 = (\lambda_1^2\nu_1^2/c^2)c^2dt^2 - (c^2/\lambda_1^2\nu_1^2)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \tag{7}$$

This line element leads to a different formulation of the refractive index as :

$$n_r = c^2/\lambda_1^2\nu_1^2 \tag{8}$$

where the subscript 'r' indicates the relativistic value. Success has therefore been achieved in treating the material medium in the same way as the gravitational field (8). A problem then became evident in the original solution of these field equations ($R_{\mu\sigma} = 0$) by Schwarzschild for the spherically symmetric gravitating body (equations 2)). That solution does not take account of the fact that a change in the (linear) velocity of EMR requires a change in its wavelength and frequency. An alternative solution (16) was found which takes this into consideration :

$$ds^2 = (1 - (\alpha\nu_r^2/c^2r))c^2dt^2 - dr^2/(1 - (\alpha\nu_r^2/c^2r)) - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{9}$$

where α is the volumetric mass and ν_r is the frequency in the presence of the gravitational field at a particular distance r. The corresponding wavelength (λ_g) is given by the equation :

$$\lambda_g = (c/\nu_r)(1 - (\alpha\nu_r^2/c^2r))^{1/2} \tag{10}$$

This solution implies that there can be no such thing as an absolute black hole since the line element is frequency dependent, so that one can always find EMR of low enough frequency (or long enough wavelength) that will be able to escape from the gravitating body whatever its mass and radius. It is also observed that from the viewpoint of a dimensional analysis, a volumetric mass is certainly preferable to a geometric one (the latter being the result of the Schwarzschild solution).

Now that both the material medium and the gravitational field have been treated separately on the same basis, the logical question is to find a combined solution that also satisfies the field equations ($R_{\mu\sigma} = 0$).

This solution was found for an isotropic material medium in a spherically symmetric gravitational field (17) :

$$ds^2 = (\lambda_1^2\nu_1^2/c^2)(1 - (\alpha\nu_{1r}^2/\lambda_1^2\nu_1^2r))c^2dt^2 - (c^2/\lambda_1^2\nu_1^2)(dr^2/(1 - (\alpha\nu_{1r}^2/\lambda_1^2\nu_1^2r)) + r^2(d\theta^2 + \sin^2\theta d\phi^2)) \tag{11}$$

where ν_{1r} is the frequency in the medium in the presence of the gravitational field at a particular distance r. The corresponding wavelength (λ_{1g}) is given by :

$$\lambda_{1g} = (\lambda_1\nu_1/\nu_{1r})(1 - (\alpha\nu_{1r}^2/\lambda_1^2\nu_1^2r))^{1/2} \tag{12}$$

Equations (10) and (12) are seen as defining the relationship between the wavelength and the frequency of EMR for free space and an isotropic material medium respectively in the presence of a spherically symmetric gravitational field. They replace the corresponding expressions ($c = \lambda\nu$ and

$v = \lambda_1 \nu_1$) which are used in formulating the classical refractive index in equation(4) and its relativistic counterpart in equation (8) and which do not take account of the gravitational field. A distinction between ν and ν_r, or ν_1 and ν_{1r} (similarly for λ and λ_g, λ_1 and λ_{1g})can only be conceived in the present space age.[18] Thus ν, ν_1, λ and λ_1 could be measured in the absence of a gravitational field (as approximated in an orbiting laboratory in free fall) while ν_r, ν_{1r}, λ_g and λ_{1g} could be values measured on the suface of a particular gravitating mass. Strictly speaking then, the quantities c, λ, ν, v, λ_1 and ν_1 have never been measured since science has until recently been restricted to a gravitational field! The values used for these quantities were measured on the earth's surface!

In a previous paper (4) equations (7), and (9) and (11) were simplified by assuming $\nu \simeq \nu_1 \simeq \nu_r \simeq \nu_{1r}$. In other words only changes in wavelength were considered. This is a first essentially practical approximation for an earthbound observer! It allowed for a reformulation (4) of the refractive index for an isotropic material medium on the surface of a spherically symmetric gravitating body :

$$n_r^{\theta,\phi} = (\lambda^2/\lambda_1^2)((1 - (\alpha/\lambda^2 r))/(1 - (\alpha/\lambda_1^2 r)))^{1/2} \qquad (13a)$$

$$n_r^r = (\lambda^2 - (\alpha/r))/(\lambda_1^2 - (\alpha/r)) \qquad (13b)$$

where the superscripts θ and ϕ indicate a measurement in the tangential plane and r a measurement in the radial direction. This approximate formulation was adequate for expaining the basic features of 'anomalous' refractive index dispersion curves.

CO-ORDINATE VELOCITY

The new formulation of the refractive index just given was made using the co-ordinate velocity of the light wave. There appears to be some confusion as to the significance of this quantity. In my opinion it expresses the velocity that an observer measures when he is located at a given co-ordinate point in a system of co-ordinates. Thus if a system of spherical polar co-ordinates with origin at the center of the earth (taken as a sphere) is considered, points on the earth's surface can be specified for given values of r, θ and ϕ . A velocity measured at such a point is a co-ordinate velocity for that system of co-ordinates.
It is made within the approximation that the small variations in r, θ and ϕ incurred in the measurement are not significant. It should be observed that a co-ordinate velocity can thus only be measured when the observer and his laboratory are 'anchored' at the given point and motion is only allowed within the laboratory.

SIGNIFICANCE OF WAVELENGTH AND FREQUENCY DEPENDENCE

The use of a wavelength and (implicitly) frequency dependent approach (or what C.N. Yang has called local phase invariance (3)) in deriving these metrics (equations 7), 9) and 11)) provides what may be termed a built in 'Uncertainty Principle' (2), since one has to state the wavelength of of EMR used in making any observation. If the wavelength is long, there is greater error in the measurement of position but less error in the measurement of momentum, and if the wavelength is short the opposite is true. The distinction between 'macroscopic' and 'microscopic' quantities is also no longer apparent since no wavelength of EMR is given any special significance. If one uses long wavelengths, 'macroscopic' quantities are observed whereas for short wavelengths, 'microscopic' quantities are observed. However, the same line element describes both types of quantities. The distinction between what is a particle and what is a medium also depends solely on the wavelength of the EMR . As an example, a closed wooden box with a metal coin inside is a particle to visible EMR which are reflected off the wood. To X-rays which penetrate the wood however, the box is a medium and the metal coin inside is a particle as the X-rays are reflected off it !

FURTHER WORK

The new formulations of the refractive index in equations (13a) and (13b) have raised many questions including the very design of refractometers. Except for a few sophisticated experiments, no consideration has generally been given by experimenters in this field to any possible directional dependence of their measurements (18). It must also be realized that this formulation is only a first approximation.
A study of expressions derived from the more general equations (7), (9) and (11) would involve measurements made in free fall as explained earlier!

Finally, it would also be interesting to find other phenomena that could be described in terms of these wavelength and frequency dependent metrics. The objective is to provide a purely field explanation (19) for these phenomena rather than taking the conventional mechanistic approach which requires the creation of new particles every time greater energy levels are achieved.

REFERENCES

1) A. EINSTEIN, H. A. LORENTZ, H. WEYL and H. MINKOWSKI; *"The Principle of Relativity"*, Dover (1952), New York, p. 37, 111, 38, 107, 75, 186.

2) A. EINSTEIN; *"Ideas and Opinions"*, Crown (Bonanza) Publishers (1954), New York, p. 285,276.

3) C.N. YANG; "*Physics Today*", 33(6), (1980), p. 42 - 46.

4) C. A. GALL "*Int. J. Theor. Phys.*", 19(12)(1980) p. 889 - 897.

5) P. G. BERGMANN, "*Introduction to the Theory of Relativity*", Dover (1976), New York, p. 44, 47, 121.

6) R. ADLER, M. BAZIN and M. SCHIFFER; " *Introduction to General Relativity*", Mc Graw Hill(1975) New York, p. 169, 185, 194, 168, 344.

7) K. SCHWARZSCHILD; "*Sitzber. Preuss. Akad.Wiss*". Berlín, (1916), p. 189 - 196.

8) A. EINSTEIN; "*The Meaning of Relativity*", Princeton Univ. Press, Princeton, N.J.,5th. edition (1979), p. 124.

9) R. G. COLLINGWOOD; "*The Idea of Nature*", Oxford Univ. Press (1978), Oxford, p. 17-19, 142 - 157

10) G. S. KIRK and J. E. RAVEN; "*The Presocratic Philosophers*", Cambridge Univ. Press (1957), p. 400- 426.

11) H. GEORGI and S. L. GLASHOW; "*Physics Today*",33 (9), (1980), p. 30.

12) M. BORN and E. WOLF; "*Principles of Optics*" Pergamon Press (1959), New York, p. 83-84, 118.

13) E. BORTOLOTTI; "*Rendiconti della classe di scienze fisiche, matematiche e naturali dell' Academia nazionale dei Lincei*", 6a., 4 (1926), p. 552.

14) R.K. LUNEBERG; "*Propagation of Electromagnetic Waves*" (mimeographed notes of lectures delivered at New York University, 1947-1948), p. 98-100.

15) C. A. GALL; "*An Isotropic Medium in the Absence of a Gravitational Field (Zero Body Approximation)*", unpublished work.

16) C. A. GALL; *Frequency and Wavelength Dependence of the Gravitational One Body Problem* " , unpublished work.

17) C. A. GALL; "*An Isotropic Medium in a Spherically Symmetric Gravitational Field (One Body Approximation)*", unpublished work.

18) A. A. MICHELSON; "*Astrophysics J.*", 61(3)(1925) p. 137 - 139; A. A. MICHELSON, H. G. GALE and F. PEARSON; ibid, p. 140 - 145.

19) A. EINSTEIN and L. INFELD; "*The Evolution of Physics*", Simon and Schuster (1966), New York, p. 240 - 245.

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