Rev. Téc. Ing., Univ. Zulia
Vol. 7, No. 2, 1984

Generalized elliptic-type integrals

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Integrals of the form [1]

$$\mu_j(k) = \int_0^{\pi} (1-k^2\cos\phi)^{-j-1/2}\, d\phi \quad (1)$$

where $0<k<1$, and j is a positive integer, occur in radiation field problems [2].
For particular values of j,(1) reduces to the complete elliptic integrals of the first and second kind. In a recent paper [3], the author has given a generalization of (1) in the following form :

$$S_\mu(k,\nu) = \int_0^{\pi} \frac{\sin^{2\nu}\phi\, d\phi}{(1-k^2\cos\phi)^{\mu+1/2}} \quad (2)$$

$0\leq k<1$, $Re(\mu)> -1/2$, $Re(\nu)> -1/2$, obtaining a series expansion and some other formulae.Kalla,Conde and Hubbell [4] have studied a family of integrals of the form

$$R_\mu(k,\alpha,\gamma) = \int_0^{\pi} \frac{\cos^{2\alpha-1}(\theta/2)\, \sin^{2\gamma-2\alpha-1}(\theta/2)\, d\theta}{(1-k^2\cos\theta)^{\mu+1/2}} \quad (3)$$

where $0\leq k<1$, $Re(\gamma)>Re(\alpha)>0$, $Re(\mu)> -1/2$.
This paper contains series expansions,its relationship with Gauss'hypergeometric function, asymptotic expansions valid in the neighborhood of $k^2=1$, and some recurrence relations. We observe that

$$R_\mu(k,\alpha,2\alpha) = 2^{1-2\alpha} S_\mu(k,\alpha-1/2) \quad (4)$$

and

$$R_j(k,1/2,1) = S_j(k,0) = \mu_j(k) \quad (5)$$

In the present note, we generalize (1),(2) and (3) as the generalized elliptic-type integrals of the following form :

$$\mu_\mu(k,\beta,\delta) = \int_0^{\pi} {}_2F_1(\mu+1/2,\beta;\delta;k^2\cos\phi)\, d\phi \quad (6)$$

$$S_\mu^{\beta,\delta}(k,\nu) = \int_0^{\pi} \sin^{2\nu}\phi \cdot {}_2F_1(\mu+1/2,\beta;\delta;k^2\cos\phi)\, d\phi \quad (7)$$

and

$$R_\mu^{\beta,\delta}(k,\alpha,\gamma) = \int_0^{\pi} \cos^{2\alpha-1}(\phi/2)\, \sin^{2\gamma-2\alpha-1}(\phi/2)\, {}_2F_1(\mu+1/2,\beta;\delta;\, k^2\cos\phi)\, d\phi \quad (8)$$

where ${}_2F_1(\alpha,\beta;\delta;z)$ is the Gauss' hypergeometric function [5]. By virtue of the identity

$${}_2F_1(\alpha,\beta;\beta;z) = (1-z)^{-\alpha} \quad , \quad |\arg(1-z)|<\pi$$

for $\beta=\delta$, (6), (7) and (8), reduces to (1) (2) and (3) respectively.

In following papers, we propose to study the generalized elliptic-type integrals (6), (7) and (8).

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Recibido el 8 de agosto de 1984