Rev. Téc. Ing. Univ. Zulia. Vol. 17, No. 1, 51-53, 1994

Boundary effects on vortex flow

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Abstract

We consider a simple model for a vortex ring in an incompressible, irrotational fluid within an infinite circular cylinder. The effects of the boundary, at which the radial derivative of the flow potential is required to vanish, on the axial flow is examined and found to be negligible as long as the cylinder radius is at least twice that of the vortex ring.

Efectos de borde en flujo vortice

Resumen

Se considera un modelo simple para un anillo vórtice en un fluido irrotacional e incompresible en el interior de un cilindro circular infinito. Se examina los efectos del borde sobre el flujo axial, en el cual se requiere que la derivada radial del flujo potencial sea cero y se encontró que es insignificante mientras sea el radio del cilindro como mínimo dos veces el radio del anillo vórtice.

The motion of vortices, a fascinating phenomenon treated in classical hydrodynamics, continues to be of interest in various modern settings. For example, the generation and flow of vortices was proposed by Feynman [1] as a mechanism for determining the critical velocity of superfluids in restricted geometries and Peshkov [2] subsequently found that super-flow data could be fit satisfactorily using classical vortex flow in an unbounded ideal fluid. In this brief note we present a simple model to examine boundary effects on the axial flow velocity of a vortex ring down the axis of an infinite cylinder. Although the results do not relate directly to vortex generation in superfluids, they do confirm Peshkov's assumption that boundary effects should not be important.

Consider a vortex ring singularity of unit radius coaxial with an infinite cylinder of radius $a > 1$. The remainder of the fluid is assumed to be irrotational and incompressible, and so is describable by a scalar flow potential u. The circulation integral about the vortex core is assumed to have the value 2, which leads to the simple boundary value problem (in cylindrical coordinates ρ, z; there is azimuthal symmetry).

$$\nabla^2 u_a = 0 \tag{1a}$$

$$u_a(\rho, z = 0^{\pm}) = \pm\,\theta(1-\rho) \tag{1b}$$

$$\partial u_a / \partial\rho(a, z) = 0. \tag{1c}$$

Also, it is assumed that u_a vanishes as $z \to \pm\infty$. Here θ denotes the unit step function. The solution to (1) is elementary [3]

$$u_a(\rho, z) = \frac{2}{a}\sum_{n=1}^{\infty} \frac{J_0(\rho j_n/a)J_1(j_n/a)}{j_n J_0^2(j_n)}\, e^{-j_n|z|/a}\,\mathrm{sgn}(z) \tag{2}$$

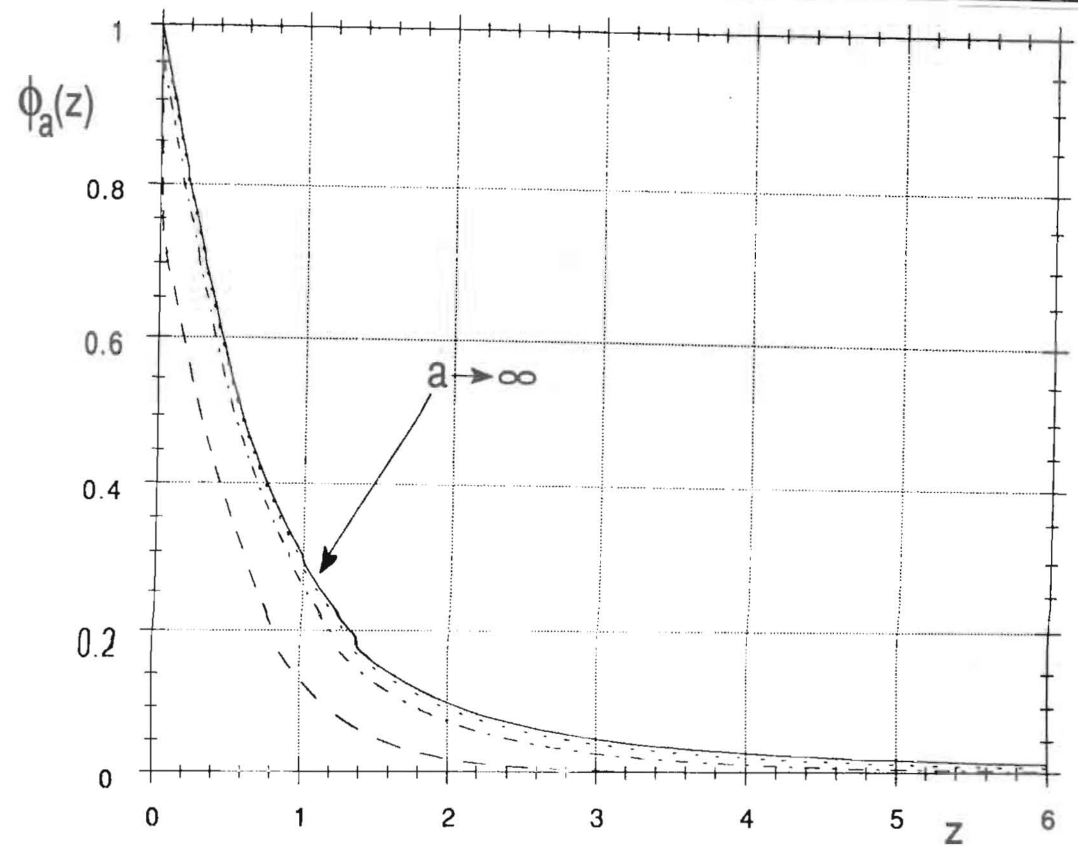

Figure 1. Flow potential along the cylinder axis. Dashed curve: a = 2; dash-dot curve: a = 5; dotted curve: a = 10. The solid curve represents Eq. (4).

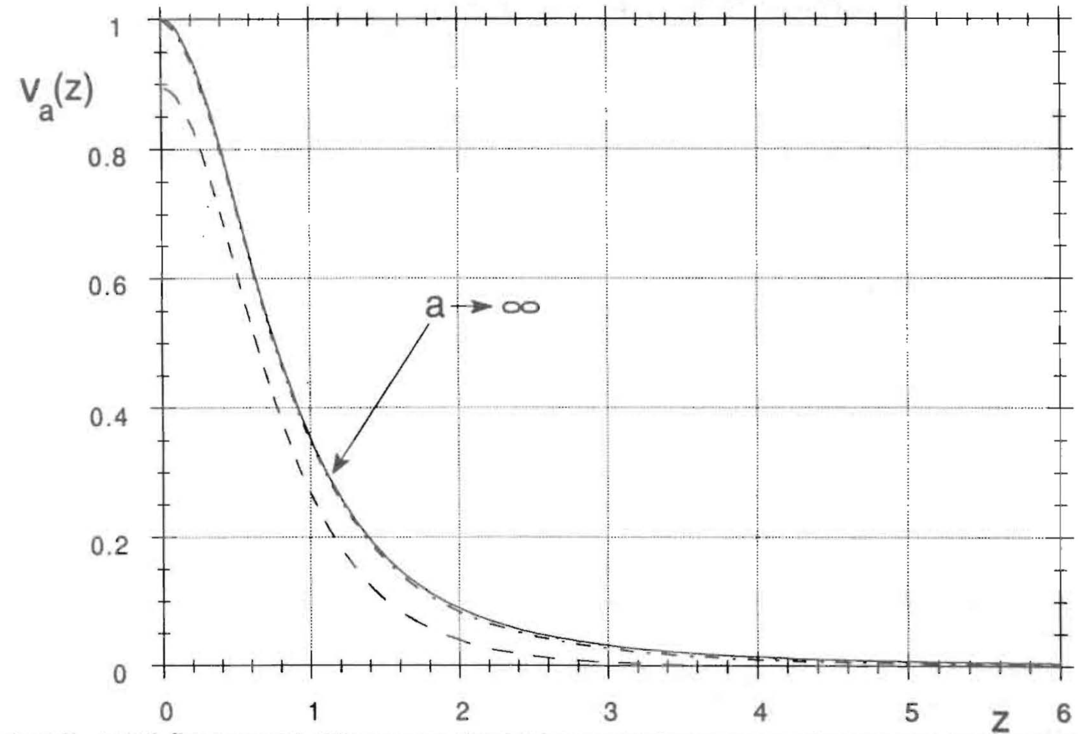

Figure 2. Axial flow speed. The curve designation is the same as for Figure 1. The dotted curve overlaps the solid curve and cannot be distinguished.

where the j_n are the positive roots of the Bessel function J_1. For the unbounded fluid, where $a \rightarrow \infty$, the solution is

$$u_\infty(\rho,z) = sgn(z) \int_0^\infty J_1(\lambda) J_0(\lambda\rho) e^{-\lambda|z|} d\lambda. \tag{3}$$

To assess the a-dependence of the flow, we look at the flow potential along the cylinder axis, $\rho = 0$. In this case,

$$\phi_\infty(z) = u_\infty(0,z) = (\sqrt{z^2+1} - z)/\sqrt{z^2+1}, \; (z > 0) \tag{4}$$

We present in Figure 1 a graph of equation (4) along with plots of $\phi_a(z) = u_a(0,z)$ for $a = 2,5,10$. The corresponding comparison is presented for the axial flow speed $v_a(z) = \partial\phi_a(z)/\partial z$ in Figure 2. It appears from these results that boundary effects are unimportant, except when the votex and channel radii are nearly the same size, and are thus unlikely to be a consideration for the experiments reviewed in [2].

Acknowledgement

This research was sponsored by the Division of Nuclear Physics, U.S. Department of Energy under Contract No. DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

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Recibido el 20 de Octubre de 1992