Stability of implicit self-tuning controllers for time-varying systems based on Lyapunov function

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Abstract

The important issue on self-tuning control includes the stability, performance and convergence of involved recursive algorithms. Based on a Lyapunov function, this paper proves the stability of implicit self-tuning controllers, combining recursive parameters estimation with a forgetting factor and generalized minimum variance criterion, for time-varying systems. The system parameters are considered to be changing continuously but slowly or changing abruptly but infrequently. The analysis is extended to the case where the system model is subject to system and measurement noises. The main results are the theorems which assure the overall stability of the closed-loop system, which are proved in a straight way compared with previous stability analysis results.

Key words: Self-tuning control, generalized minimum variance control, sliding-mode control, discrete-time systems, time-varying systems, Lyapunov function.

Estabilidad de controladores auto-ajustables para sistemas variantes en el tiempo basada en funciones Lyapunov

Resumen

Los problemas más importantes de los controladores auto-ajustables, se refieren a la estabilidad, desempeño y convergencia de los algoritmos recursivos involucrados. Basándose en funciones Lyapunov, este trabajo prueba la estabilidad de los controladores auto-ajustables implícitos, combinando la estimación recursiva de los parámetros del controlador incluyendo el factor de olvido con el criterio de variancia mínima generalizada, aplicados a los sistemas variantes en el tiempo. Se considera que los parámetros del sistema cambian continua pero lentamente o cambian abrupta pero infrecuentemente. El análisis también se extiende al caso donde el modelo del sistema incluye ruido. Los principales resultados presentados en este trabajo son los teoremas que aseguran la estabilidad global del sistema en lazo cerrado, los cuales son probados en una manera mucho más directa comparando con resultados previos en el área.

Palabras clave: controladores auto-ajustables, control de varianza mínima generalizada, control por régimen deslizante, sistemas discretos, sistemas variantes en el tiempo, función de Lyapunov.

1. Introduction

Åström [1] showed, in a stochastic way, that a self-tuning minimum variance control is optimal for a general CARMA (Controlled Auto-Regressive Moving Average) model, even though the noise parameters are not explicitly estimated; however the proof begins with the assumptions that parameters convergence is assured (not necessarily to the optimal values) and the system model is minimum phase.

By extending the minimum variance criterion (MVC) of [1], Clarke [2] propose the generalized minimum variance control (GMVC) for non-minimum phase systems by the use of a cost function which incorporates system input and set-point variation, and a control law was derived for a system model with known parameters. The parameters of the control law for the real systems with unknown parameters are estimated using a recursive least-squares (RLS) algorithm. Furuta [3] proposed a discrete-time variable structure system (VSS) approach to the case where system parameters are unknown; the VSS is designed based on MVC or GMVC, and a recursive estimator of controller parameters is applied.

Based on key technical lemmas, the global convergence of implicit self-tuning controllers was studied for discrete-time minimum phase linear systems in a seminal paper by Goodwin [4] and for explicit self-tuning controllers in the case of non-minimum phase systems by Goodwin [5]. From the viewpoint of sliding mode control (SMC), Patete [6, 7] gave a complete proof for the stability of implicit self-tuning controllers based on GMVC for minimum or non-minimum phase systems by the use of a Lyapunov function. However, all these researches have been done for time-invariant systems (TIS). Clark [8] studied the stability of self-tuning controllers for time-invariant systems subject to noise, based on the idea of describing the system in a feedback form and using the notion of dissipative-real systems. However no rigorous stability proof was given.

The purpose of this paper is to analyze the stability of the implicit self-tuning controller for discrete time-varying systems (TVS) and discrete time-varying systems subject to system and measurement noises. The criterion considered is the minimization of an auxiliary controlled variable based on the concept of sliding mode control to yield the system stability.

The paper is organized as follows: in section 2, the GMVC based on the sliding mode control concept [3] is reviewed. Section 3 studies the recursive estimation of controller parameters based on GMVC to deal with time-varying systems. Simulation examples are given in section 4. Some remarks conclude the paper.

2. Generalized minimum variance control

The controller design with the GMVC based on the sliding mode control concept, in the case of time-invariant systems [3, 6, 7], is reviewed in this section. The discrete-time single-input single-output (SISO) time-invariant system is considered. The representation of the nominal system with input u_k and output y_k is given by:

$$A^0(z^{-1})y_k = z^{-d}B^0(z^{-1})u_k, \qquad (1)$$

where $A^0(z^{-1})$ and $B^0(z^{-1})$ have no common factors and z denotes the time-shift operator $z^{-1}y_k = y_{k-t}$. In the Laplace transformation, the time-shift operator is described as $z = e^{sT_0}$ where T_0 is the sampling period (for simplicity, and without loss of generality, $T_0 = 1$ is assumed). In order to derive the nominal control law the polynomials $A^0(z^{-1})$ and $B^0(z^{-1})$ are assumed to have constant and known parameters, represented by: $A^0(z^{-1})y_k = 1 + a_1z^{-1} + \ldots + a_nz^{-n}$ and $B^0(z^{-1})y_k = 1 + b_1z^{-1} + \ldots + b_mz^{-m}$, where $b_0 \neq 0$. The delay step, d, is also assumed to be known. The control objective is to minimize the variance of the controlled variables s_{k+d}, defined in the deterministic case as:

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k, \qquad (2)$$

where the polynomials $C(z^{-1})y_k = 1 + c_1z^{-1} + c_2z^{-2} + \ldots + c_nz^{-n}$ and $Q(z^{-1}) = q_0(1 - z^{-1})$ are to be designed, so that the specifications written below should be satisfied. The error signal e_k is defined as $e_k = y_k - r_k$, where r_k is the reference signal.

The idea is similar to the discrete-time sliding mode control [3].

The polynomial $C(z^{-1})$ is chosen Schur and should be designed by assigning all characteristic roots inside the unit disk in the z-plane. Equation (2) is rewritten as:

$$s_{k+d} = G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+d}, \quad (3)$$

where the polynomial $G(z^{-1})$ is defined as $G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1})$, i.e. $G(z^{-1}) = g_0 + g_1 z^{-1} + \ldots + g_{m+d-1} z^{m+d-1}$ and the polynomials $E(z^{-1}) = e_0 + e_1 z^{-1} + \ldots + e_{d-1} z^{-(d-1)}$ and $F(z^{-1}) = f_0 + f_1 z^{-1} + \ldots + f_{n-1} z^{-(n-1)}$ satisfy the equality:

$$C(z^{-1}) = A^0(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}). \quad (4)$$

Then the GMVC input required to vanish s_{k+d} in Eq. (2) is given by:

$$u_k = -G(z^{-1})^{-1}\left[F(z^{-1})y_k - C(z^{-1})r_{k+d}\right], \quad (5)$$

where the polynomials $C(z^{-1})$ and $Q(z^{-1})$ are chosen to make the control system satisfy the following Lemma.

Lemma [9]: The necessary and sufficient condition for the control input to make $s_{k+d} = 0$ stable is that all the roots of the polynomial $T^0(z^{-1})$:

$$T^0(z^{-1}) = A^0(z^{-1})Q(z^{-1}) + B^0(z^{-1})C(z^{-1}) = 0, \quad (6)$$

belong to the open unit disk, and the polynomials (Q, C), (A^0, C), (B^0, Q) have no common zeros outside of the unit disk.

The uncertainty in system characteristics leads to a certain family of models rather than to a single system model to be considered. In the case where the uncertainties come from parametric perturbations, we have a family of closed-loop characteristic polynomials $T(z^{-1})$ instead of a single nominal characteristic polynomial $T^0(z^{-1})$. Defining $q = z^{-1} = e^{-j\omega}$, which maps the stable zone inside the unit circle into the outside in the z-plane, then $T(z^{-1})$ is defined as:

$$T(q) = T^0(q) + \gamma\rho T(q), \quad (7)$$

where γ is a positive constant representing the margin of perturbation and $\rho T(q) \in \Omega$ gives a set of admissible perturbations defined as $\beta(\omega) = set\left(\gamma\rho T(e^{-j\omega})\,|0 \le \omega \le 2\pi\right)$. For robust stability analysis, we may use the method by Tsypkin [10] for closed-loop discrete-time systems, which involves the modified characteristic locus criterion.

Criterion [10]: For robust stability of closed-loop discrete-time parametric systems, it is sufficient that

$$\tilde{T}(e^{-j\omega}) = \frac{T^0(e^{-j\omega})}{\left|Q(e^{-jw})\right|\sum_{i=0}^{n}\alpha_i + \left|C(e^{-j\omega})\right|\sum_{l=0}^{m}\beta_l}, \quad (8)$$

does not enclose and does not intersect the critical circle o of radio γ and centered at the origin, i.e. $o(\gamma,0)$, when ω moves from 0 to 2π. α_i and β_l are the range of parametric perturbations, defined as $\alpha_i = \dfrac{\overline{a}_i - \underline{a}_i}{2\gamma}$, $\beta_l = \dfrac{\overline{b}_l - \underline{b}_l}{2\gamma}$. Where $\overline{a}_i, \overline{b}_l$ and $\underline{a}_i, \underline{b}_l$ are the upper and lower bounds of a_i and b_l, respectively.

3. Self-tuning control of time-varying systems based on GMVC

It has been proved [6, 7] that the following self-tuning algorithm assures the overall stability for SISO time invariant systems, when the system constant parameters are not accurately known, by the recursive estimation of the controller parameters $F(z^{-1})$ and $G(z^{-1})$, under the following assumptions.

Assumptions 1: a) The order of the system in Eq. (1) is known. b) The delay step d is known. c) Polynomial $C(z^{-1})$ is Schur. d) Lemma 1 is satisfied. e) Criterion 1 is satisfied. f) The given reference signal r is bounded.

The self-tuning control based on GMVC algorithm is given by the following recursive estimation equations:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\Gamma_{k-1}\phi_{k-d}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}} \times \left[s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-1}\right], \quad (9),$$

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\Gamma_{k-1}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}} \quad (10)$$

where $\phi_k^T = [y_k, y_{k-1}, \ldots, y_{k-n+1}, u_k, \ldots, u_{k-m-d+1}]$ is the vector containing measured output and control signal data. $\theta^T = [f_0, f_1, f_{n-1}, g_0, g_1, g_{m+d-1}]$ is the vector containing the parameters of $F(z^{-1})$ and $G(z^{-1})$, and $\hat{\theta}^T = [\hat{f}_0, \hat{f}_1, \hat{f}_{n-1}, \hat{g}_0, \hat{g}_1, \hat{g}_{m+d-1}]$ is the estimate of θ.

Then the controller includes identified parameters as follows:

$$u_k = -\hat{G}(z^{-1})^{-1}\left[\hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d}\right], \quad (11)$$

where $\hat{F}(z^{-1})$ and $\hat{G}(z^{-1})$ are estimates of $F(z^{-1})$ and $G(z^{-1})$, respectively.

In several adaptive problems it is of interest to consider the situation in which the parameters are time-varying. From now on, system parameters are assumed to change abruptly but infrequently or changing continuously but slowly. Then, the family of system models is represented by:

$$A(z^{-1})y_k = z^{-d}B(z^{-1})u_k, \quad (12)$$

where (1) is the nominal system model of (12). The self-tuning control based on GMVC algorithm presented is extended to the case where a forgetting factor is introduced into the recursive estimate equations (9) and (10) to deal with this type of time-varying systems.

Assumptions 2: a) The unknown time-varying system parameters are assumed all uniformly bounded away from infinity. b) The time-varying controller parameters vector θ_k has the following model, $\theta_k = \theta_{k-1} + \eta_k$, where η_k is a zero mean time-varying signal. Thus, $E\{\theta_k\} = E\{\theta_{k-1}\}$ with E_η denoting the expectation with respect to η_k.

Theorem 1

Recursive estimates of controller parameters based on generalized minimum variance criterion with a forgetting factor: Given a positive definite matrix Γ_0, a parameter $\mu(0 < \mu < 1)$ and an initial parameters vector $\hat{\theta}_0$, if the estimate $\hat{\theta}_k$ of the controller in Eq. (11) is given by the recursive equations:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\Gamma_{k-1}\phi_{k-d}}{\mu+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}} \times \left[s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-1}\right], \quad (13)$$

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\Gamma_{k-1}}{\mu+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}}, \quad (14)$$

under Assumptions 1 and 2, then the overall closed-loop time-varying system combining equations (11), (13), (14) and equation (12) gives the overall stability in the sense of the expectation with respect to η_k.

Proof: Using the control law in Eq. (11), s_{k+d} may be rewritten as:

$$s_{k+d} = \phi_k^T\tilde{\theta}_{k-d}, \quad (15)$$

where,

$$\tilde{\theta}_k = \theta - \hat{\theta}_k. \quad (16)$$

The candidate Lyapunov function is given by:

$$E_\eta[\Delta V_k] = E_\eta\left\{\frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k\right\}. \quad (17)$$

The time difference of Eq. (17) is considered as follows:

$$E_\eta[\Delta V_k] = E_\eta\{V_k - \mu V_{k-1}\}, \quad (18)$$

where μ is the forgetting factor, $0 < \mu < 1$. Then, for ΔV_k, the following is derived:

$$\Delta V_k = \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2\mu + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}\mu \quad (19)$$

$$\Delta V_k = -\frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^T \Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1})\mu + \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2\mu + \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} + \Gamma_{k-1}^{-1}\mu)\tilde{\theta}_k - \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}\mu \quad (20)$$

$$\Delta V_k = -\frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^T \Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1})\mu + s_k^2 + \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} + \Gamma_{k-1}^{-1}\mu)\tilde{\theta}_k - \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}\mu + \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_k\mu - \frac{1}{2}s_{k-1}^2\mu - \frac{1}{2}s_k^2 . \quad (21)$$

From Eq. (15), s_k^2 is:

$$s_k^2 = \tilde{\theta}_k^T\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k. \quad (22)$$

Substituting Eq. (22) into Eq. (21), the following relation can be obtained:

$$\Delta V_k = -\frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^T \Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1})\mu + \frac{1}{2}s_{k-1}^2\mu + \frac{1}{2}\tilde{\theta}_k^T(\Gamma_k^{-1} - \Gamma_{k-1}^{-1}\mu - \phi_{k-d}\phi_{k-d}^T)\tilde{\theta}_k + \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\mu(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k\mu^{-1}) \quad (23)$$

The third and fourth terms in the right-hand side of Eq. (23) may be equal to zero as follows:

From the third term on the right-hand side of Eq. (23),

$$\Gamma_k^{-1} - \Gamma_{k-1}^{-1}\mu - \phi_{k-d}\phi_{k-d}^T = 0, \quad (24)$$

$$\Gamma_k = (\Gamma_{k-1}^{-1}\mu - \phi_{k-d}\phi_{k-d}^T)^{-1}. \quad (25)$$

Equation (25) yields Eq. (14) by the matrix inversion lemma. From the fourth term of Eq. (23):

$$\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\mu^{-1} = 0. \quad (26)$$

Using Eq. (3) and Eq. (16), and substituting them into Eq. (26) the following is obtained:

$$\theta_k = \hat{\theta}_{k-1} + \mu^{-1}\Gamma_{k-1}\phi_{k-d}(s_k - \phi_{k-d}^T\hat{\theta}_k + C(z^{-1})r_k) + \theta_k - \theta_{k-1}, \quad (27)$$

$$(\mu + \phi_{k-d}^T\Gamma_{k-1}\phi_{k-d})\hat{\theta}_k = (\mu + \phi_{k-d}^T\Gamma_{k-1}\phi_{k-d})\hat{\theta}_{k-1} + \mu\theta_k - \mu\theta_{k-1} + \Gamma_{k-1}\phi_{k-d}(s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-1}). \quad (28)$$

Finally, by taking the expectation with respect to η_k in (28), Eq. (13) is derived.

Then, by using of the recursive equations (13) and (14) for a positive bounded V_0, $E_\eta\{\Delta V_k\}$ is proved negative semi-definite, i.e. $E_\eta\{\Delta V_k\} \le 0$, as follows:

Using the recursive equations (13) and (14) into (23), for $k = 1$ the following relation is obtained

$$E_\eta\{V_1 - \mu V_0\} = E_\eta\left\{-\frac{1}{2}s_0^2\mu - \frac{1}{2}(\tilde{\theta}_1 - \tilde{\theta}_0)^T\Gamma_0^{-1}(\tilde{\theta}_1 - \tilde{\theta}_0)\mu\right\}. \quad (29)$$

Initially $\tilde{\theta}_1 - \tilde{\theta}_0 \neq 0$, then $E_\eta\{V_1 - \mu V_0\} < 0$, which gives $E_\eta\{V_1\} < E_\eta\{\mu V_0\}$. For $k = 2$:

$$E_\eta\left\{V_2 + \frac{1}{2}s_1^2\mu + \frac{1}{2}(\tilde{\theta}_2 - \tilde{\theta}_1)^T\Gamma_1^{-1}(\tilde{\theta}_2 - \tilde{\theta}_1)\mu\right\} = E_\eta\{\mu V_1\} < E_\eta\{\mu^2 V_0\}. \quad (30)$$

Then, for a large N the following relation is derived:

$$E_\eta\left\{V_N + \frac{1}{2}\sum_{k=2}^{N}\left(s_{k-1}^2 + (\tilde{\theta}_k - \tilde{\theta}_{k-1})^T\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1})\right) \times \mu^{N-k+1}\right\} < E_\eta\{\mu^N V_0\}. \quad (31)$$

Equation (31) implies that μ^N approaches to zero as N goes to infinity; then the left-hand side of Eq. (31) will also vanish. Thus, s_N and $(\tilde{\theta}_N - \tilde{\theta}_{N-1})$ vanish as N approaches to infinity.

Since the polynomial $Q(z^{-1})$ is designed to satisfy Eq. (6) and Eq. (7) for a bounded reference r_k, both input u_k and output y_k are shown to be bounded, i.e. multiplying Eq. (2) by $A(z^{-1})$ and using Eq. (12), the following expression for the control signal u_k is derived:

$$u_k = \frac{A(z^{-1})}{A(z^{-1})Q(z^{-1}) + C(z^{-1})B(z^{-1})} s_{k+d} + \frac{A(z^{-1})C(z^{-1})}{A(z^{-1})Q(z^{-1}) + C(z^{-1})B(z^{-1})} r_{k+d}. \quad (32)$$

s_{k+d} vanishes in the sense of expectation with respect to η_k as k goes to infinity, as shown in Eq. (31). Since Eq. (7) is designed robust stable then, for a bounded reference r_k, u_k is proved to be bounded for all k; thus from Eq. (2) y_k is bounded, furthermore e_k are shown bounded for all k, and the overall stability of the closed- loop time-varying system is assured in the sense of expectation with respect to η_k. Especially for a constant reference r_k, when s_k approaches to zero in the sense of expectation with respect to η_k as k approaches to infinity, from Eq. (32) the control signal u_k approaches to a constant, i.e. $u_k = u_{k-1}$. This implies that, $Q(z^{-1})u_k = 0$. Then, from Eq. (2), the output signal y_k approaches to r_k; furthermore the error signal e_k approaches to zero, and the output signal y_k convergence to the constant reference signal r_k is assured in the sense of expectation with respect to η_k.■

Remarks: The usage of a parameter μ in the difference of the Lyapunov function Eq. (18) is similar to the introduction of a forgetting factor in the least-squares error function [9], which implies that a time-varying weighting of the data is introduced. The most recent data is given unit weight, but data that is t time units old is weighted by μ^t.

We do not prove, or claim, that $\hat{\theta}_k$ converges to its true values θ. Instead, each element of $\hat{\theta}_k - \hat{\theta}_{k-1}$ approaches to constant values in the sense of expectation with respect to η_k.

In general, real systems are also subject to noise, and it is of interest to ensure the overall closed-loop stability in presence of system and measurement noises. The proposed algorithm is extended to the case where system and measurement noises are considered. The white noise signal ξ_k is defined as a bounded independent random sequence, which has the following properties:

$$\begin{array}{l} \mathrm{E}_{\xi}\{\xi_i\} = 0 \\ \mathrm{E}_{\xi}\{\xi_j \xi_i\} = \delta_{ij}\sigma^2 \end{array} \quad \text{with} \quad \delta_{ij} = \begin{cases} 0 \text{ for } i \neq j \\ 1 \text{ for } i = j \end{cases},$$

where ξ_k is a zero mean uncorrelated random signal with standard deviation σ and E_{ξ} is the expectation with respect to noise ξ_k.

The nominal system model and the family of system models to be considered in this section are represented as:

$$A^0(z^{-1})y_k = z^{-d}B^0(z^{-1})u_k + \xi_k, \quad (33)$$

$$A(z^{-1})y_k = z^{-d}B(z^{-1})u_k + \xi_k, \quad (34)$$

respectively, where ξ_k represents the system and measurement noise (white noise) signal. This model is so-called AR (Auto Regressive) model.

Using the definition of s_{k+d} given in Eq. (2), and including equations (4) and (33), s_{k+d} is rewritten as

$$s_{k+d} = G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+ed} + E(z^{-1})\xi_{k+d}, \quad (35)$$

If the control law in Eq. (11) is used for the exactly known system, then:

$$s_{k+d} = E(z^{-1})\xi_{k+d}. \quad (36)$$

The degree of polynomial $E(z^{-1})$ is $d-1$, which implies that s_{k+d} depends only on future states of ξ. Therefore, u_k gives the minimum variance control for s_{k+d}.

Theorem 2

Recursive estimates of controller parameters based on generalized minimum variance criterion for auto regressive system models with a forgetting factor: Given a positive definite matrix Γ_0, a parameter μ ($0 < \mu < 1$) and the initial parameters vector $\hat{\theta}_0$, the estimate $\hat{\theta}_k$ of the controller in Eq. (11) satisfies the recursive equations (13) and (14) for a white zero mean noise under Assumptions 1 and 2; thus the overall closed-loop time-varying system combining equations (11), (13), (14) and equation (34) gives the overall stability in the sense of the expectation with respect to system and measurement noise ξ_k and with respect to η_k.

Proof: The proof follows as the given proof in Theorem 1, using equations (33)-(36), combined with the proof given in Patete [11]-Theorem 2 for auto regressive time-invariant systems.■

4. Simulation results

In the following, the proposed algorithm in Eq. (13) and Eq. (14) is denoted by STC-TVS-GMVC. The initial condition $\Gamma_0 = I$ (where I is the identity matrix) and the parameter $\mu = 0.8$ (forgetting factor) are chosen. The reference signal is set to the unit step.

As an academic example, consider the following non-minimum phase system model with d = 2:

$$(1 + a_1 z^{-1}) y_k = z^{-d}(b_0 + b_1 z^{-1}) u_k, \tag{37}$$

where the parameter intervals are given as $a_1 \in [-0.5, -0.1]$, $b_0 \in [0.8, 1.2]$, $b_1 \in [1.4, 2.6]$. For $\gamma = 1$

$$a_i^0 = \frac{\overline{a}_i + \underline{a}_i}{2},\ \alpha_i^0 = \frac{\overline{a}_i - \underline{a}_i}{2\gamma},\ b_l^0 = \frac{\overline{b}_l + \underline{b}_l}{2},$$
$$\beta_l^0 = \frac{\overline{b}_l - \underline{b}_l}{2\gamma}$$

thus, the nominal system model is represented by:

$$(1 - 0.3z^{-1}) y_k = z^{-d}(1 + 2z^{-1}) u_k, \tag{38}$$

and $\alpha_1 = 0.2$, $\beta_0 = 0.2$, $\beta_1 = 0.6$.

For the nominal controller design using the GMVC presented in section 2, the following polynomials are chosen:

$$C(z^{-1}) = 1 + z^{-1} + 0.25z^{-2}, \tag{39}$$

$$Q(z^{-1}) = 40(1 + z^{-1}), \tag{40}$$

which lead to the following polynomials for the controller law: $\hat{F}(z^{-1}) = 0.64$ and $\hat{G}(z^{-1}) = 41 - 36.7z^{-1} + 2.6z^{-2}$, these are used as initial estimates of the controller parameters.

In the first, for the simulation example, the real system model is assumed to be represented by:

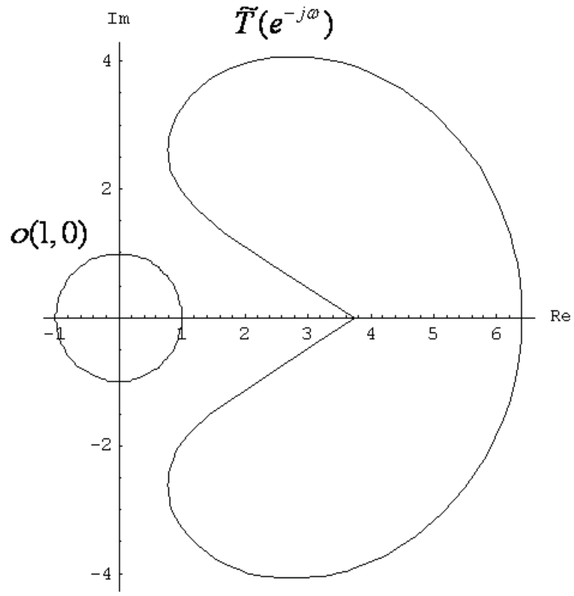

Figure 1. Robust stability analysis for system (37): $\tilde{T}(e^{-j\omega})$, $o(\gamma,0)$ and $\gamma = 1$.

$$(1 - 0.5z^{-1}) y_k = z^{-d}(1 + 2.3z^{-1}) u_k. \tag{41}$$

The robust stability analysis of the closed-loop system in presence of parametric interval uncertainties is shown in Figure 1. As shown, $\tilde{T}(e^{-j\omega})$ does not intersect with the critical circle $o(1,0)$, which implies that the sufficient condition for robust stability is satisfied. Figure 2 shows the output responses when, after 100 samples, the system model in Eq. (41) abruptly changes to the following system model:

$$(1 - 0.1z^{-1}) y_k = z^{-d}(1.2 + 2.6z^{-1}) u_k. \tag{42}$$

5. Conclusions

The overall stability of a self-tuning control algorithm, based on recursive controller parameter estimation including a forgetting factor and generalized minimum variance criterion, for a class of time-varying systems, has been proved based on the discrete-time sliding mode control theory. The results have been extended to the case where system and measurement noises are considered into the system model. The validity of the proposed algorithm was also demonstrated through simulation results. The principal contribution of the obtained stability results is to as-

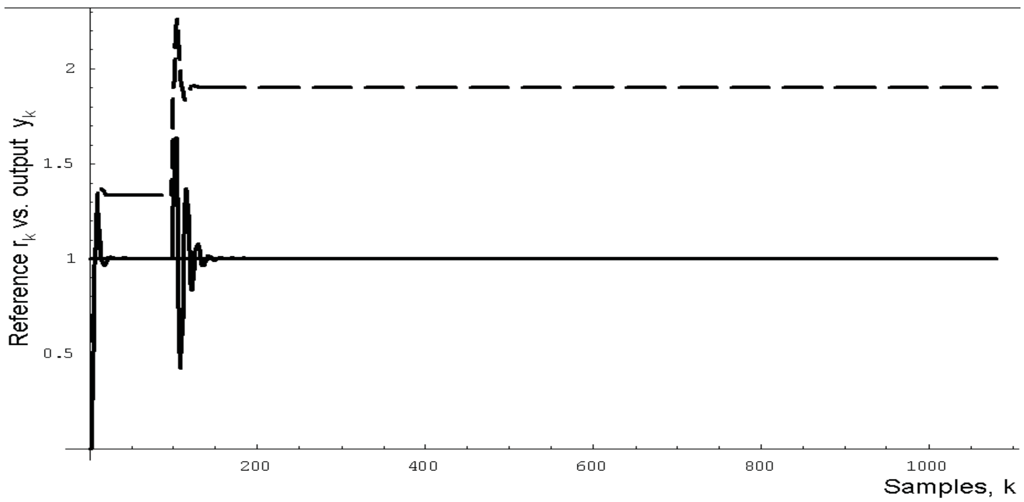

Figure 2. y_k vs. r_k: GMVC (dashed-line) and STC-TVS-GMVC (solid-line) algorithms applied to system in Eq. (37), when after 100 samples the system changes from Eq. (41) to Eq. (42), $\Gamma = I$ and $\mu = 0.8$.

sure the overall stability if the presented control algorithm is implemented on a real system with time-varying parameters, even in the presence of system and measurement noises.

Acknowledgments

This research was supported by the COE Grant in Aid of MEXT, Japan. The authors acknowledge the comments and suggestions given by Dr. Akihiko Sugiki of Tokyo Denki University, Japan.

References

1. Astrom, K. and Wittenmark B.: "On Self-Tuning Regulators", Automatica, 9 (1973) 185-199.
2. Clarke, D., Phil D. and Gawthrop P.: "Self-Tuning Controller", IEE Proc., Vol. 122, No. 9 (1975) 929-934
3. Furuta, K.: "VSS Type Self-Tuning Control", IEEE Trans. Industrial Electronics, Vol. 40, No. 4 (1993) 37-44.
4. Goodwin, G., Ramadge, P. and Caines P.: "Discrete-Time Multivariable Adaptive Control", IEEE Transactions on Automatic Control Vol. 25, No. 3 (1980) 449-456.
5. Goodwin, G. and Sin, K.: "Adaptive Control of Nonminimum Phases Systems", IEEE Transactions on Automatic Control, Vol. 26, No. 2 (1981) 478-483.
6. Patete, A., Furuta, K. and Tomizuka, M.: "Stability of Self-Tuning Control Based on Lyapunov Function", International Journal of Adaptive Control and Signal Processing, Vol. 22, No 8 (2008) 795-810.
7. Patete, A., Furuta, K. and Tomizuka, M.: "Self-Tuning Control Based on Generalized Minimum Variance Criterion", IFAC Workshop on Adaptation and Learning in Control and Signal Processing (ALCOSP'07), Saint Petersburg, Russia, August, (2007).
8. Clarke, D., Phil D., and Gawhrop P.: "Self-Tuning Control", IEE Proceedings, Vol. 126 No. 6 (1979) 633-640.
9. Astrom, K. and Wittenmark, B.: "Adaptive Control", Addison-Wesley, (1989).
10. Tsypkin, Y. and Furuta, K.: "Frequency-Domain Criteria of Robust Stability: Discrete-Time and Continuous-Time Systems, A unified Approach", International Journal of Robust and Nonlinear Control, Vol. 5 (1995) 207-222.
11. Patete, A., Furuta, K. and Tomizuka, M.: "Self-tuning control based on Generalized Minimum Variance Criterion for Auto Regressive Models", Automatica, Vol. 44, No. 8 (2007) 1970-1975.

Recibido el 8 de Diciembre de 2009
En forma revisada el 10 de Enero de 2011